„DUNĂREA DE JOS" UNIVERSITY OF GALAȚI

The School for Doctoral Studies in Mechanical and Industrial Engineering

SUMMARY
DOCTORAL THESIS

CONTRIBUTIONS TO IMPROVE CONSTRUCTION SAFETY OF THE SHIPS IN THE CONTEXT OF INTERNATIONAL AND NATIONAL REGULATIONS

PhD Student,

Eng. Dumitru LUPAŞCU

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Series I6: Mechanical engineering No. 41

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FOREWORD

The development of the PhD thesis and the elaboration of the PhD thesis I made at the School for Doctoral Studies in Engineering within "Dunarea de Jos" University of Galati, under the scientific direction of Prof. Univ. PhD. Eng. Ionel CHIRICĂ, to whom I thank and bring my deep recognition in this way for the competent and generous scientific guidance that he has given me in this professional and research work that has allowed me to finalize this thesis.

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Finally, I pay tribute to my parents, Neculai LUPAŞCU and Maria LUPAŞCU, who, through immense efforts and sacrifices, grew me up, educated and supported me to become a good naval architect.

Eng. Dumitru LUPAŞCU
Galaţi, 2018

CUPRINS

INTRODUCTION

Construction safety of the ships is a central objective in the concerns of all the actors involved in shipbuilding and operation of ships, this being subject to the integrity of ships, the life of crews and passengers, the integrity of the goods transported, and the environment protection. The safety construction is achieved by meeting some technical requirements from international and national regulations as well as from the classification societies rules and industry standards, in all phases of the ship's life, from design, construction and operation to scrapping.

Therefore, the purpose of this paper is the analysis of some technical requirements concerning the construction safety of the ships, stipulated by the main international and national regulations and following theoretical and laboratory research, to make proposals for improvement of these requirements, thus contributing to technical progress in the field.

In this respect it has been established the following objectives:

1. Development of computational tools for assessing the construction safety of the ships on longitudinal strength;
2. Improving the requirements on longitudinal strength of seagoing ships in intact condition;
3. Development of a method of probabilistic assessment of longitudinal residual strength of the seagoing ships in damaged condition;
4. Development of a method of probabilistic assessment of overall survival of the seagoing ships in damaged condition.

The paper is structured in 6 chapters presenting the current state of the technical requirements and calculation methods, and the novelties and own contributions of the author, as appropriate, as follows:

Chapter 1. Safety construction as provided in international and national regulations. In this chapter is made a presentation of the international and national regulations which ensures the construction safety of the ships and is briefly described how this safety is achieved in the design, construction and operation phase.

Chapter 2. Computational tools designed and used to analyze the construction safety of the ships. To analyze the construction safety of the ships, the following 4 computational programs which allow conducting studies and research to improve shipbuilding regulations on safety construction were developed:

- A program for calculation of the sectional efforts and of the elastic line of the ship’s hull at the static position in still-water and at the quasi-static position on the head wave having cosine or trochoidal form. The program allows determining the floating parameters for the equilibrium position of the ship in still-water and the quasi-static position on the wave for different cases of loading conditions, as well as the sectional efforts and the elastic line of its hull under such conditions. Based on the classical calculation method in "Ship’s Theory," adapted and developed by the author in a specific way to automate calculations, the program introduces several innovative elements described in this chapter.
- A program for linear calculation of the ship’s oscillation parameters and of the additional sectional efforts in its hull induced by the head wave. The program was developed based on the classical method in „Ship’s Theory”, using the „Ordinary

Strip Theory" and the „Modified Strip Theory" developed by the author in a specific way to automate calculations, introducing several new considerations. The tests on a series of 3 models validated the adopted method and the calculation program.

- A program for non-linear calculation of the ship's oscillation parameters and of the additional sectional efforts in its hull induced by the head wave. The program was developed based on the classical method in „Ship's Theory", using the „Ordinary strip theory" and the „Modified strip theory" developed by the author in a specific way to automate calculations, introducing several new considerations. The tests on a series of 2 models validated the adopted method and the calculation program.
- A program for non-linear calculation of the ship's oscillation parameters and of the additional sectional efforts in its hull induced by head waves, considering quadratic damping. The program was developed based on a new method proposed by the author. The tests on a series of 2 models validated the adopted method and the calculation program.

Chapter 3. Improving the construction safety of the ships regarding longitudinal strength in intact condition. Following the comparison of wave-induced sectional efforts, determined according to IACS regulations, with those obtained with the programs presented in Chapter 2, it was found that the IACS efforts are smaller than the real ones, particularly in the case of shear forces for several representative ship types. This situation leads to the realization of undersized vessels confirmed by the loss of many bulk carriers of single side skin construction. To avoid such incidents, appropriate corrections are proposed for IACS formulations to obtain the real values for the sectional efforts and the construction of properly dimensioned ships that are safer in operation.

Chapter 4. Probabilistic assessment of longitudinal residual strength of the seagoing damaged ships. To improve the safety of the construction of the damaged ship, it is proposed to apply a new probabilistic concept for longitudinal residual strength assessment, which is based on survival after damage, as a measure of the safety assessment of the ship in damaged conditions, hereinafter referred to as the effective longitudinal residual strength R_L index. For the survival of the ship, it is required that this effective index be greater than a minimum value called the required longitudinal residual strength index R_{Lo}.

Chapter 5. Probabilistic assessment of overall survival of the seagoing damaged ships. To improve the construction safety of the damaged ship, it is proposed to complement the SOLAS probability concept for assessing the stability of the damaged ship, with survival and in terms of the hull ultimate girder strength after failure.

Chapter 6. General conclusions, original contributions and perspectives. This chapter presents the personal contributions to improving construction safety of the ships within the thesis and the conclusions drawn from the application of these contributions as well as potential research directions related to the approached topic which will be developed in the future.

1 CONSTRUCTION SAFETY OF THE SHIPS AS PROVIDED IN INTERNATIONAL AND NATIONAL REGULATIONS

1.1 The importance of the construction safety of the ships

The objective of this paper is, following an analysis of some of the requirements concerning construction safety of the ships under the main international and national regulations and after a theoretical and laboratory research, to contribute to their improvement.

The importance of these requirements is underlined by the fact that one of the important objectives of the International Maritime Organization (IMO) (Figure 1.1.1) and other international bodies is the safety of vessels for their operation without human or material loss and without environmental pollution.

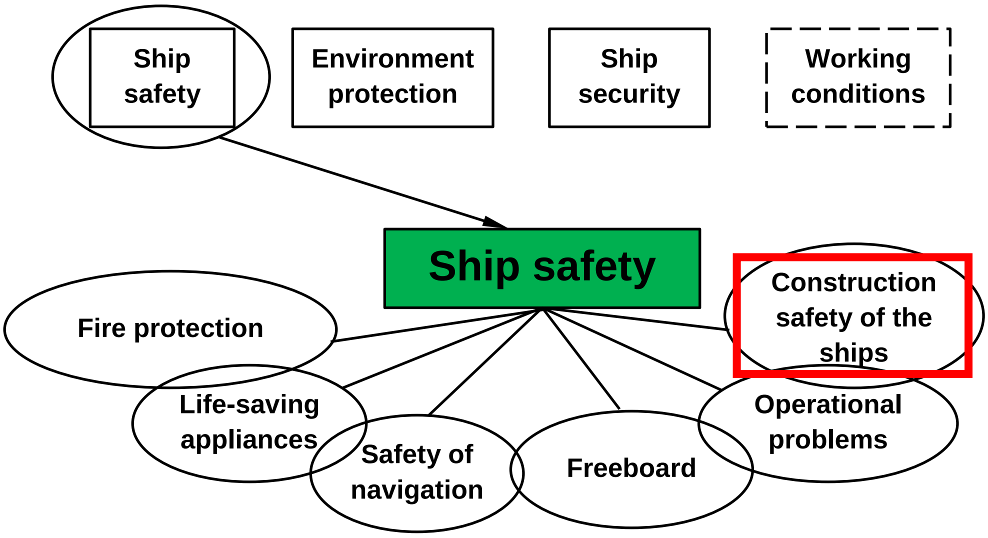


Fig. 1.1.1 – IMO goals presented in an unitary approach

This safety includes the safety construction as an essential element, which is achieved through a set of activities carried out by researchers, designers and shipbuilders in accordance with international regulations, classification societies rules and technical norms of naval authorities regarding hull construction, subdivision and stability of ships, construction of machinery and installations, fire protection.

For this purpose, it has been decided within the IMO that this organization should exercise its attributions by ship construction standards based on its goals shown in Fig. 1.1.1.

The first three tiers were achieved within the IMO, by adopting the resolution MSC.296(87) [2] on the Guidelines for verification of conformity with goal-based ship construction standards for bulk carriers and oil tankers and, by resolution MSC.290(87) [3], the new SOLAS regulations II-1/2.28 and II-1/3-10 [4], and IV and V tiers were achieved by the International Association of Classification Societies (IACS) by developing the Common Structural Rules for Bulk Carriers and Double Hull Oil Tanker [5], but the process of developing and improvement of these 5 tiers is continuous consistent with the technical progress and the needs of maritime transport.

Similarly, it is envisaged the safety of the construction of inland waterway vessels, this being ensured by the regulations of international bodies such as the Danube Commission, the Central Commission for Navigation on the Rhine, the UNECE, the European Commission, etc. as well as by the rules of the naval authorities and classification societies.

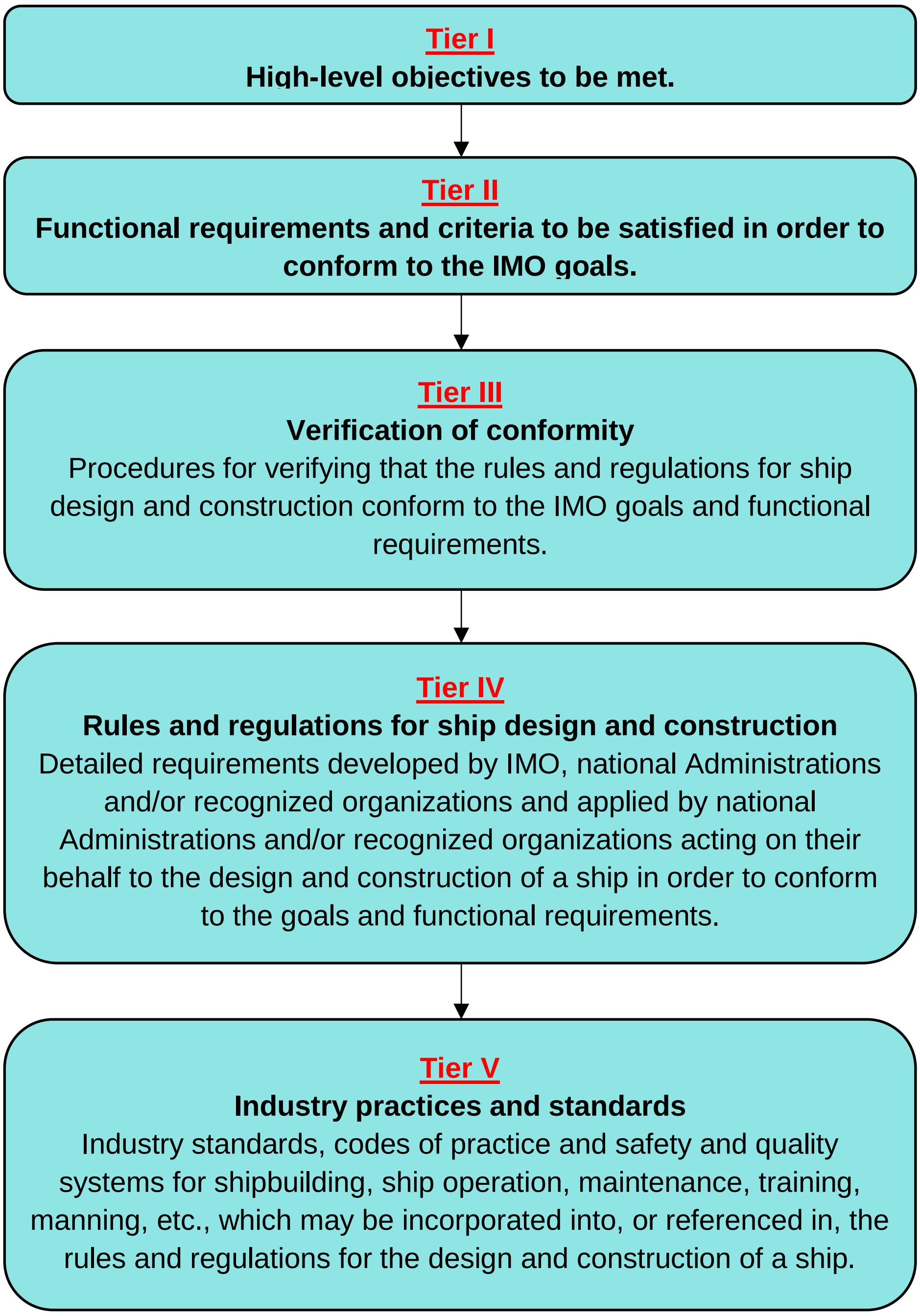


Fig. 1.1.2 – Goal-based system (GBS) organization

A lower level of safety construction can lead to material losses and failures, human life losses or ecological disasters.

Among the lost ships, a significant share was represented by the bulk carriers and oil tankers, some of them by the collapse of the longitudinal structural elements of the hull and its breaking into two parts, as can be seen in Fig. 1.1.3 [8], 1.1.4 [9] and 1.1.5 [10].

Studies have shown that losses caused by hull breaking depend in a probabilistic manner on the following factors:

- the ship's age, increasing with it;
- the type of cargo, increasing with its specific gravity;
- the chose route, the most dangerous routes being in the Far East and the Northern Atlantic;
- the type of used material, high strength steels increasing the risk of the ship being lost because they are more susceptible to corrosion, having an less corrosion addition, and assuring a greater elasticity to the hull, fostering the appearance of the «springing» phenomenon, i.e. the appearance of the ship general vibration induced by waves, which additionally stress it and weakens its strength to fatigue.

Fig. 1.1.3 – EUROBULKER X Bulk Carrier after breaking, on 02.09.2000, while loading cement at the port of Lefkandi in Greece [8]

Fig. 1.1.4 – ERIKA oil tanker after breaking, on 12 December 1999, at 60 miles from the coast of Brittany [9]

Fig. 1.1.5 – PRESTIGE oil tanker after breaking on 13 November 2002, at 30 miles from the North-East coast of Spain[10]

1.2 International and national regulations on the construction safety of the ships

The construction safety of seagoing ships is achieved by applying of the international and national regulations requirements on general and local strength, and the buoyancy and stability.

At International level, are in force the regulations developed by the IMO, IACS and classification societies. At national level, are in force the regulations developed by the Romanian Naval Authority (RNA):

At European level, for inland waterway vessels are in force regulations developed within the European Union, the United Nations Economic Commission for Europe (UNECE), the Rhine Commission, and the Danube Commission. At national level, the regulations developed by RNA are in force.

1.3 Assessment of the ships safety construction

The assessment of the construction safety of the ships is begun at the design stage by verifying the fulfillment of the strength criteria as well as the buoyancy and stability criteria in the design documentation by a naval authority and/or by a recognized classification society.

During the construction phase, the ship strength is verified by the quality assurance compartments in shipyard and by the naval authority whose flag the ship is flying and/or by a recognized classification society through technical survey carried out to ascertain that the approved design and the manufacturing technologies are in accordance with the regulations in force:

The assessment of the safety construction of ships in service is conducted on periodical surveys carried out at 5 years interval, of intermediate surveys carried out at 2.5 years interval and of annual surveys by the naval authority and/or by recognized classification society.

2 COMPUTATIONAL TOOLS DESIGNED AND USED TO ANALYZE THE CONSTRUCTION SAFETY OF THE SHIPS

2.1 Overview

To analyze the construction safety of the ships, the following 4 computational programs which allow conducting studies and research to improve shipbuilding regulations on safety construction were developed by author.

2.2 A program for calculation of the sectional efforts and of the elastic line of the ship's hull at the static position in still-water and at the quasi-static position on the head wave.

2.2.1 Program object and destination

The program allows determining the floating parameters for the equilibrium position of the ship in still-water and for the quasi-static position on the head wave having cosine or trochoidal form, for different cases of loading conditions, as well as the sectional efforts and the elastic line of its hull under such conditions.

2.2.2 The applied calculation method

Determination of the floating parameters for the equilibrium position of the ship in still-water and for the quasi-static layout on the wave for different cases of operational loading conditions, as well as the sectional efforts and the elastic line of its hull are performed principally according to the classic method from papers [48] to [57] in the "Ship Theory," "Ship Computation and Construction" and "Resistance of Materials" fields and developed and completed by the author in a specific manner, to the level of detail required for the programming of the calculation.

For this purpose, the ship's hull surface described by points is related to an orthogonal axis system (Figure 2.2.1). The points are arranged on the theoretical couples located along the ship length and define their profile by draft and half-breadth.

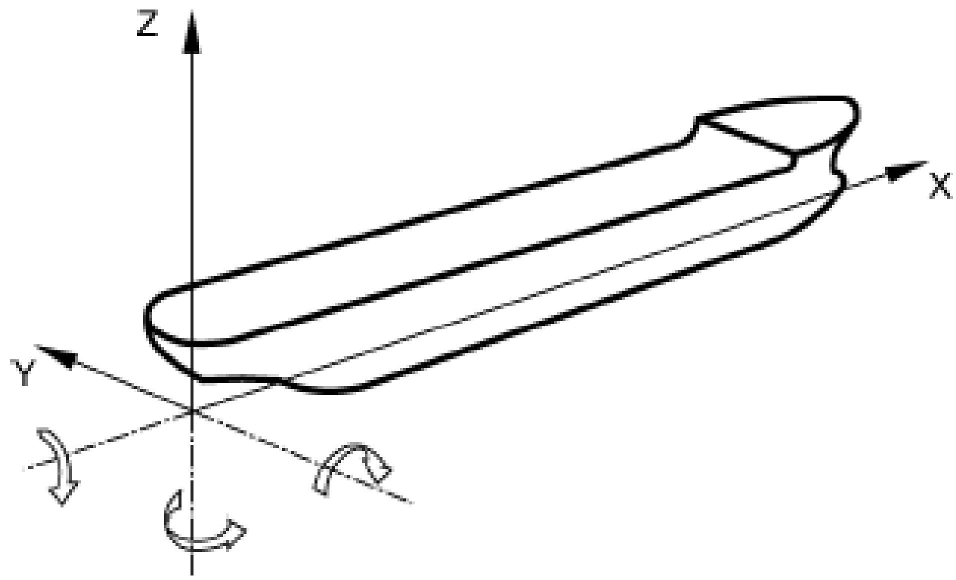


Fig. 2.2.1 – The reference co-ordinate system to which the hull surface relates [5]

The ship is considered to be in the equilibrium on still-water or in the quasi-static position on the wave, if its displacement is equal with the buoyancy force of the immersed hull and if its center of gravity G is on the same vertical with center of buoyancy C (Figure 2.2.2), which means that the following relations are fulfilled:

$$\Delta = \gamma \cdot k_a \cdot V_c \tag{2.2.1}$$

$$„ x_C - x_G = (z_G - z_C) \cdot tg\psi \tag{2.2.2}$$

$$y_C - y_G = (z_G - z_C) \cdot tg\theta \tag{2.2.3}$$

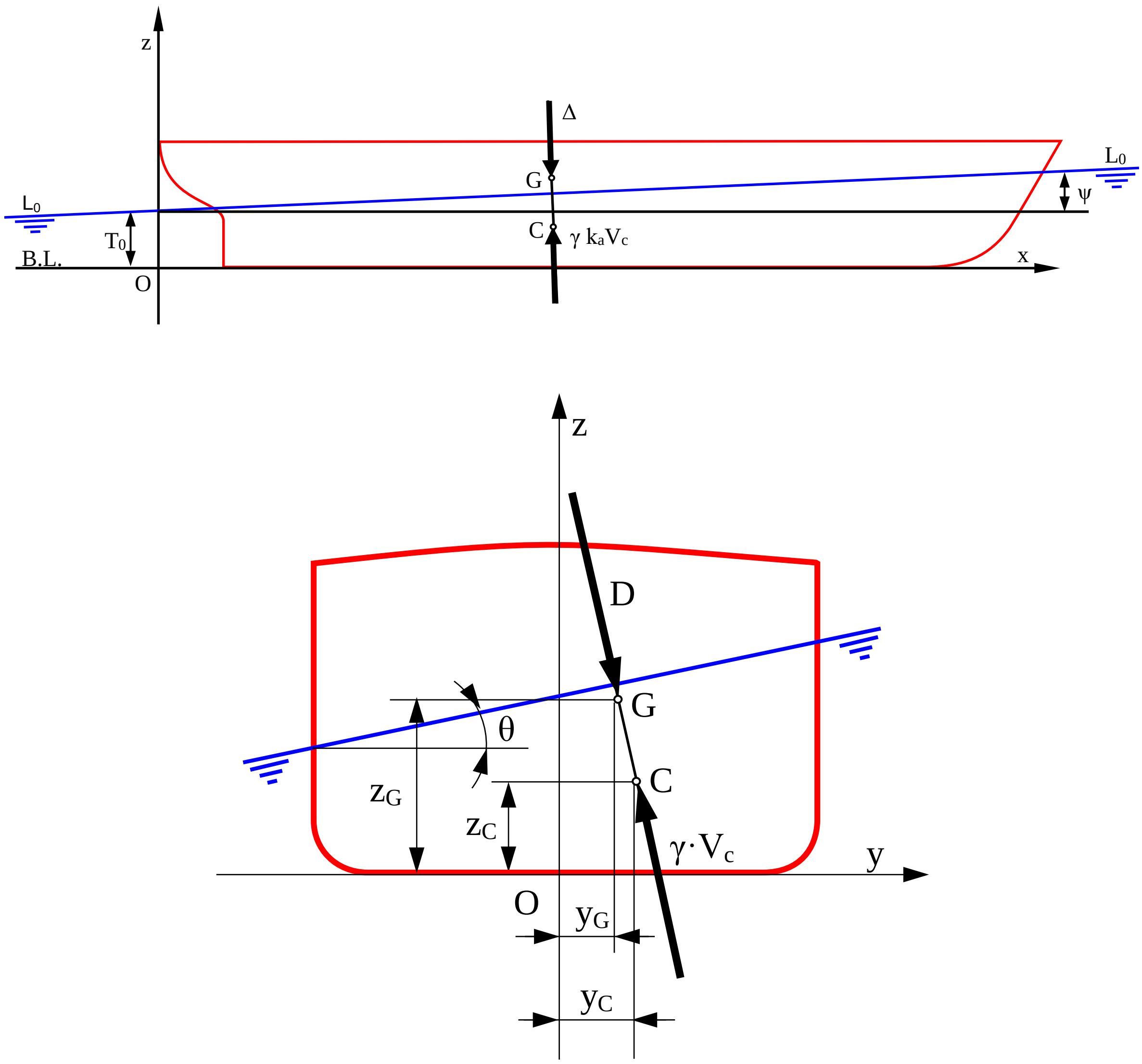


Fig. 2.2.2 – Equilibrium condition of the ship

where:

x_G – longitudinal coordinate of ship’s center of gravity;
y_G – the distance from the PD to the ship’s center of gravity;
z_G – vertical coordinate of the ship’s center of gravity;
ψ – the trim angle;
θ – the angle of longitudinal inclination (the roll or heel angle);

T_o – draught at aft ship's extremity:

γ – the specific gravity of water;

k_a – appendages coefficient (k_a = 1,001 ... 1,005)

V_C – volume of immersed hull;

x_C – abscissa of the center of buoyancy;

y_C – transverse coordinate of the center of buoyancy;

z_C – vertical coordinate of the center of buoyancy;

2. Equilibrium condition is obtained by passing two stages:

1. the longitudinal equilibration, which consists in bringing the ship to the position in which its displacement is equal to the weight of the water displaced by the immersed hull and its center of gravity G, is on the same vertical as the hull center C, which means that the relations (2.2.1) and (2.2.2) are fulfilled.

2. after longitudinal equilibration, the roll angle of the ship is determined by the relation:

$$\theta = \frac{y_G}{GM_T} \tag{2.2.4}$$

where:

GM_T – transverse metacentric height corresponding to the ship's longitudinal equilibrium position;

For ship laying on still-water, the draught *T(x)* from centerline (fig. 2.2.2) in a transversal section *x*, is given by the relation:

$$T(x) = T_o + x \cdot \psi \tag{2.2.5}$$

For ship quasi-static layout on the wave, it is theoretical and experimental proved the fact that additional vertical sectional efforts reach the maximum value when the top or hollow of the wave are amidships and the wave length is equal with the length of the ship [49].

Considering the ship quasi-static layout on the cosine wave, the draught T (x) in a transversal *x*, is given by the relations:

- for crest amidships (hogging condition) (fig. 2.2.3):

$$T(x) = T_o + x \cdot \psi - \frac{h_V}{2} \cdot \cos\frac{2 \cdot \pi \cdot x}{L} \tag{2.2.6}$$

- for hollow amidships (sagging condition) (fig. 2.2.4):

$$T(x) = T_o + x \cdot \psi + \frac{h_V}{2} \cdot \cos\frac{2 \cdot \pi \cdot x}{L} \tag{2.2.7}$$

Considering the ship quasi-static layout on the trochoidal wave, the draught *T(x)* in a transversal section *x*, is given by the relations:

- for the ship on the crest, the crest being amidships (hogging condition) (fig. 2.2.3):

$$T(x) = T_o + x \cdot \psi - \frac{h_V}{2} \cdot \cos\Phi \tag{2.2.8}$$

- for the ship on the hollow, the hollow of the wave being amidships (sagging condition) (fig. 2.2.4):

$$T(x) = T_o + x \cdot \psi + \frac{h_V}{2} \cdot \cos\Phi \tag{2.2.9}$$

where:

Φ – is a parameter obtained from transcendental equation:

$$x = \frac{\lambda_V}{2 \cdot \pi} \cdot \Phi + \frac{h}{2} \cdot \sin \Phi \qquad (2.2.10)$$

λ_V – wave length;

h_V –height of the equivalent wave corrected due to the Smith effect.

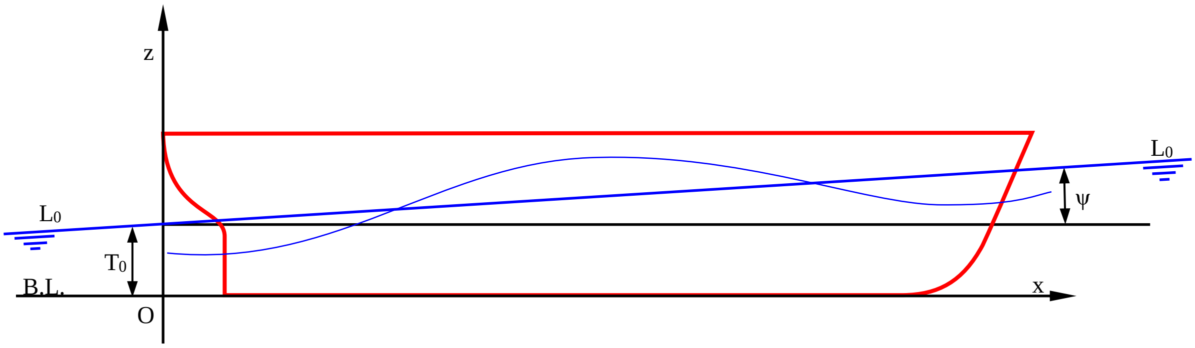


Fig. 2.2.3 – Ship quasi-static layout on wave on hogging condition

The sectional efforts are obtained by the formulas:

- vertical shear force along the ship length:

$$Q(x) = \int_0^x [m(\xi) - \gamma \cdot k_a \cdot \Omega(\xi)] \cdot d\xi \qquad (2.2.11)$$

- vertical bending moment along the ship length:

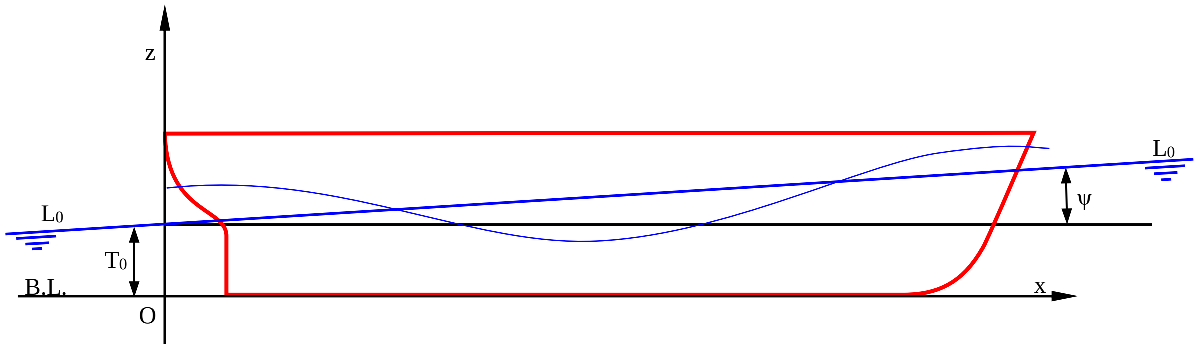


Fig. 2.2.4 – Ship quasi-static layout in wave on sagging condition.

$$M(x) = \int_0^x Q(\xi) \cdot d\xi \qquad (2.2.12)$$

- torsional moment along the ship length:

$$M_t(x) = \int_0^x [(m(\xi) \cdot y_m(\xi) - \gamma \cdot k_a \cdot B(\xi)] \cdot \cos\theta + [(m(\xi) \cdot z_m(\xi) - \gamma \cdot k_a \cdot C(\xi)] \cdot \sin\theta +$$

$$+ z_{CT}(\xi) \cdot [m(\xi) - \gamma \cdot k_a \cdot \Omega(\xi)] \cdot \sin\theta \cdot d\xi \qquad (2.2.13)$$

where:

$\Omega(\xi)$ – immersed area of the couple from ξ; section

$B(\xi)$ – static moment related to center line, of submerged area of the couple from ξ section;

$C(\xi)$ – static moment related to base line, of submerged area of the couple from ξ section
θ – roll angle (angle of list or angle of heel)
z_{CT} – vertical coordinate of the torsion center in current section;

The elastic line of the hull is determined with the relation:

- bending deflection along the ship length:

$$v(x) = v_o + \varphi_o \cdot x - \frac{1}{E} \cdot \int_0^x \int_0^x \frac{M(\xi)}{I_y(\xi)} \cdot d\xi \cdot d\xi - \frac{1}{G} \cdot \int_0^x \frac{Q(\xi)}{A_{yf}(\xi)} \cdot d\xi \qquad (2.2.14)$$

- bending rotation along the ship length:

$$\varphi(x) = \varphi_o - \frac{1}{E} \cdot \int_0^x \frac{M(\xi)}{I_y(\xi)} \cdot d\xi - \frac{Q(x)}{G \cdot A_{yf}(x)} \qquad (2.2.15)$$

In the above relations were used the notations:

I_y – moment of inertia to the horizontal neutral axis of the current cross-section of the hull girder;
A_{yf} – the shear-resistant area in the vertical direction of the current cross-section of the hull girder;
E – the longitudinal elastic modulus;
G – the transverse elastic modulus;
v_o – bending arrow at the aft end of the ship;
φ_o – bending rotation at the aft end of the ship;

The elastic line of the hull is considered to be given by the line resulting from the intersection of the deformed keel, with PD, relative to the theoretical base line (fig.2.2.5).

To determine the elastic line of the ship, in the vertical plane bending, it starts from the relation (2.2.14), in which v_o and φ_o appear as parameters to be determined from the end conditions.

In setting these parameters, the sectional efforts are determined by initially considering the ship to be a rigid body and that the deflections at the extremities are zero:

$$v_o = 0 \qquad (2.2.16)$$

$$v_l = 0 \qquad (2.2.17)$$

From these conditions it follows:

$$\varphi_o = \frac{1}{E \cdot L} \cdot \int_0^L \int_0^x \frac{M(\xi)}{I_y(\xi)} \cdot d\xi \cdot dx + \frac{1}{G \cdot L} \cdot \int_0^L \frac{Q(x)}{A_{yf}(x)} \cdot dx \qquad (2.2.18)$$

and the elastic line of the hull is determined by the relation (2.2.14).

With the deflection so calculated, the proper correction of draughts is made and the ship is balanced in still-water, thus finding the deflections at the ends of the elastic ship, relative to the ship considered as rigid body:

$$v_{pp} = T_{Epp} - T_{pp} \qquad (2.2.19)$$

$$v_{pv} = T_{Epv} - T_{pv} \qquad (2.2.20)$$

where:

T_{Epp} – the draught at the aft extremity when the ship is considered as elastic body;
T_{pp} – the draught at the aft extremity when the ship is considered as rigid body;
T_{Epv} – the draught at the forward extremity when the ship is considered as elastic body;

T_{pv} – the draught at the forward extremity when the ship is considered as rigid body;

With the new values of the draughts are determined the bending moments and the shear forces, and then the new elastic line of the body. This interactive process continues until, in two successive iterations, the peak bending moments differ by less than 0.1%.

Because the ship's body section varies longitudinally, the numerical method described below, known as the transfer matrix method, was applied to calculate the sectional stresses and vertical deformations.

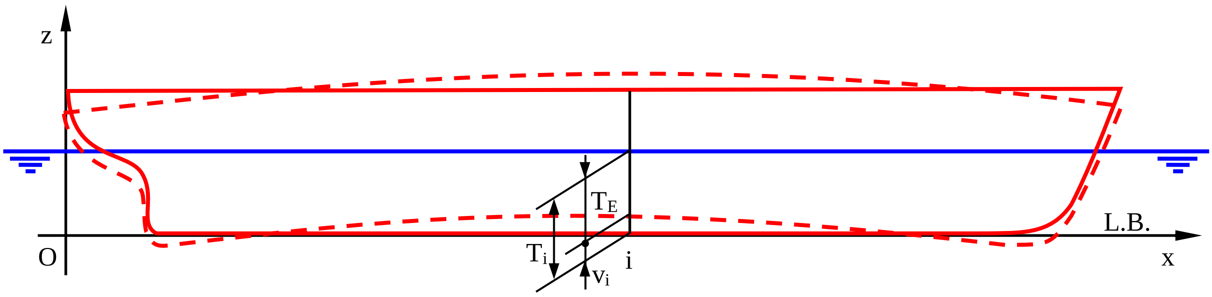


Fig. 2.2.5 – Vertically deformation of the ship's hull

The hull is considered to be a flexible thin-walled girder of variable section, meshed in elementary beams of constant section, having the length of a frame spacing (Fig.2.2.6).

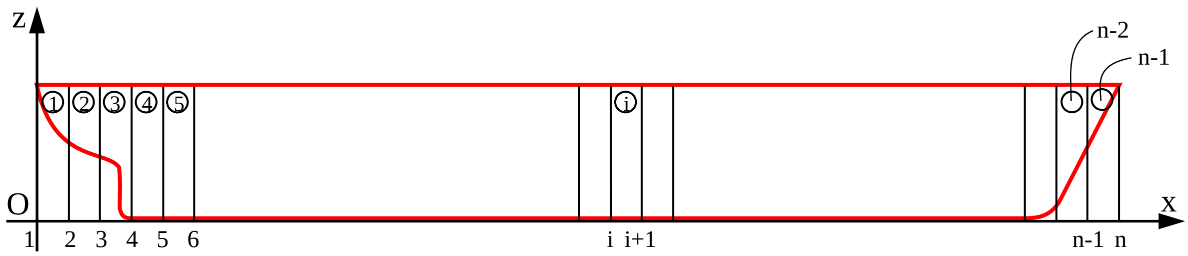


Fig. 2.2.6 – Hull girder meshed in elementary beams

By isolating of an elementary beam "i" from the ship's hull (figure 2.2.7) and bringing together the equations of equilibrium of the loads and of the sectional efforts, as well as the relationships between the deformations and the efforts for this beam, is obtained the following matrix equation which establishes the link between the efforts and the deformations at the ends of that elementary beam:

$$\begin{bmatrix} Q_{i+1} \\ M_{i+1} \\ \varphi_{i+1} \\ v_{i+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \Delta x_i & 1 & 0 & 0 \\ -\dfrac{\Delta x_i^2}{2EI_{yi}} - \dfrac{1}{GA_{yfi}} & -\dfrac{\Delta x_i}{EI_{yi}} & 1 & 0 \\ -\dfrac{\Delta x_i^3}{6EI_{yi}} - \dfrac{\Delta x_i}{GA_{yfi}} & -\dfrac{\Delta x_i^2}{2EI_{yi}} & \Delta x_i & 1 \end{bmatrix} \cdot \begin{bmatrix} Q_i \\ M_i \\ \varphi_i \\ v_i \end{bmatrix} + S_i \cdot \begin{bmatrix} \Delta x_i \\ \dfrac{\Delta x_i^2}{2} \\ -(\dfrac{\Delta x_i^2}{6EI_{yi}} + \dfrac{1}{GA_{yfi}}) \cdot \Delta x_i \\ -(\dfrac{\Delta x_i^2}{12EI_{yi}} + \dfrac{1}{GA_{yfi}}) \cdot \dfrac{\Delta x_i^2}{2} \end{bmatrix} + (S_{i+1} - S_i)/\Delta x_i \cdot \begin{bmatrix} \dfrac{\Delta x_i^2}{2} \\ \dfrac{\Delta x_i^3}{6} \\ -(\dfrac{\Delta x_i^2}{12EI_{yi}} + \dfrac{1}{GA_{yfi}}) \cdot \dfrac{\Delta x_i^2}{2} \\ -(\dfrac{\Delta x_i^2}{20EI_{yi}} + \dfrac{1}{GA_{yfi}}) \cdot \dfrac{\Delta x_i^3}{6} \end{bmatrix} \quad (2.2.21)$$

where:

Δx_i – the length of the elementary beam *"i"* situated between the coast *"i"* and *"i+1"*;

S_i – the outer load at the aft end of the elementary beam *"i":*

$$S_i = m_i - \gamma \cdot k_a \cdot \Omega_i \tag{2.2.22}$$

S_{i+1} – the outer load at the forward end of the elementary beam *"i"*:

$$S_{i+1} = m_{i+1} - \gamma \cdot k_a \cdot \Omega_{i+1}. \tag{2.2.23}$$

where:

Ω_i - the immersed area of the transverse section at the end *i*;

Ω_{i+1} - the immersed area of the transverse section at the end *i+1*.

In the aforementioned relation it was assumed that the external load varies linearly along the elementary beam *"i".*

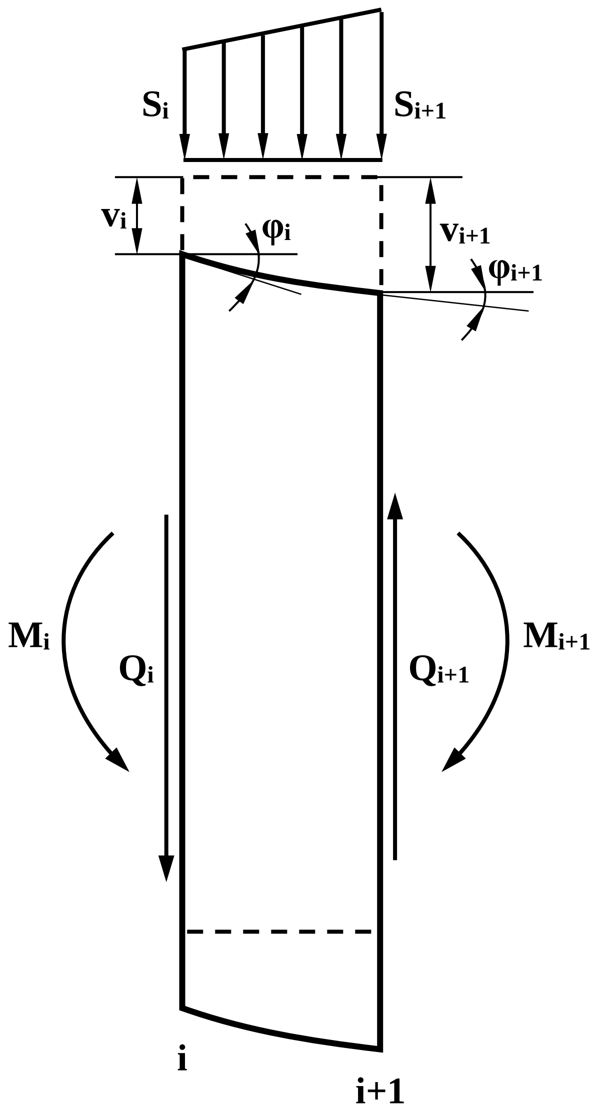


Fig. 2.2.7 – Loads, efforts and deformations in an elementary beam

Noting that:

$$\bar{v}_i = \begin{bmatrix} Q_i \\ M_i \\ \varphi_i \\ v_i \end{bmatrix} \quad (2.2.24)$$

$$[A_i] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \Delta x_i & 1 & 0 & 0 \\ -\frac{\Delta x_i^2}{2EI_{yi}} - \frac{1}{GA_{yfi}} & -\frac{\Delta x_i}{EI_{yi}} & 1 & 0 \\ -\frac{\Delta x_i^3}{6EI_{yi}} - \frac{\Delta x_i}{GA_{yfi}} & -\frac{\Delta x_i^2}{2EI_{yi}} & \Delta x_i & 1 \end{bmatrix} \quad (2.2.25)$$

$$\bar{R}_i = S_i \cdot \begin{bmatrix} \Delta x_i \\ \frac{\Delta x_i^2}{2} \\ -(\frac{\Delta x_i^2}{6EI_{yi}} + \frac{1}{GA_{yfi}}) \cdot \Delta x_i \\ -(\frac{\Delta x_i^2}{12EI_{yi}} + \frac{1}{GA_{yfi}}) \cdot \frac{\Delta x_i^2}{2} \end{bmatrix} + (S_{i+1} - S_i)/\Delta x_i \cdot \begin{bmatrix} \frac{\Delta x_i^2}{2} \\ \frac{\Delta x_i^3}{6} \\ -(\frac{\Delta x_i^2}{12EI_{yi}} + \frac{1}{GA_{yfi}}) \cdot \frac{\Delta x_i^2}{2} \\ -(\frac{\Delta x_i^2}{20EI_{yi}} + \frac{1}{GA_{yfi}}) \cdot \frac{\Delta x_i^3}{6} \end{bmatrix} \quad (2.2.26)$$

the relation (2.2.21) becomes:

$$\bar{v}_{i+1} = [A_i] \cdot \bar{v}_i + \bar{R}_i \quad (2.2.27)$$

By specifying the relation (2.2.27) for the first and the second elementary beam:

$$\bar{v}_2 = [A_1] \cdot \bar{v}_1 + \bar{R}_1 \quad (2.2.28)$$

$$\bar{v}_3 = [A_2] \cdot \bar{v}_2 + \bar{R}_2 = [A_2] \cdot [A_1] \cdot \bar{v}_1 + [A_2] \cdot \bar{R}_1 + \bar{R}_2 \quad (2.2.29)$$

or noting that:

$$[D_2] = [A_2] \cdot [A_1] \quad (2.2.30)$$

$$\bar{P}_2 = [A_2] \cdot \bar{R}_1 + \bar{R}_2 \quad (2.2.31)$$

is obtained:

$$\bar{v}_3 = [D_2] \cdot \bar{v}_1 + \bar{P}_2 \quad (2.2.32)$$

By generalization the relations (2.2.30), (2.2.31) and (2.2.32), the efforts and deformations vector at the frame "i+1" is determined:

$$\bar{v}_{i+1} = [D_i] \cdot \bar{v}_1 + \bar{P}_i \quad (2.2.33)$$

where:

$$[D_i] = [A_i] \cdot [A_{i-1}] \ldots [A_2] \cdot [A_1] \quad (2.2.34)$$

$$\bar{P}_i = [A_i] \cdot \bar{R}_{i-1} + \bar{R}_i \quad (2.2.35)$$

By applying the relation (2.2.33) to the elementary beam from the bow, the relationship between the aft and forward efforts and deformations is obtained:

$$\bar{v}_n = [D_{n-1}] \cdot \bar{v}_1 + \bar{P}_{n-1} \quad (2.2.36)$$

The ends of the ship being free, the sectional efforts are null at the *"1"* and *"n"* couples:

$Q_1 = 0$
$M_1 = 0$
$Q_n = 0$
$M_n = 0$.

There are 4 deformations and rotations of the ship's ends based on which the elastic line of the hull can be determined and for their calculation, there are only the last two equations of the system (2.2.36), so that in the first stage, it is considered that the ship's deflections at the ends are zero:

$v_1 = 0$
$v_n = 0$.

From the $v_n = 0$ condition, the aft end rotation, φ_1 is determined and then, by applying the relations (2.2.33) and (2.2.34), the sectional efforts and deformations of the hull along the ship length are determined.

With the deflections values so calculated, the proper correction of the draughts is made and the ship is in a steady position in still water.

The elastic line of the hull is determined by the relation (fig. 2.2.5):

$$v_i = T_{Ei} - T_i \tag{2.2.37}$$

where:

T_{Ei} – the draught at the frame „i” when the ship is considered as elastic body;
T_i – the draught at the frame „i” when the ship is considered as rigid body;

With the new values of the draughts of the deformed ship are determined the bending moments and the shear forces, and then the new elastic line of the body. This numerical procedure is cyclically repeated until, in two consecutive iterations, the maximum bending moments differ by less than 0.1%

2.2.3 Program description

Based on the calculation method presented in 2.2.2, the RLS-V1 program has been developed, whose code being written in the Visual-FORTRAN language that can be run on 32 or 64-bit computers running Windows XP operating system or a later version.

2.2.4 The verification of the RLS-V1 program

The verification of the program was carried out with a parallelepiped barge, with a uniform distribution of the masses along its length of 100 t/m, having the following main characteristics:

L = 100,00 m
B = 20,00 m
D = 20,00 m
T = 5,00 m
γ = 1,025 t/m^3
I_y = 20,000 m^4
A_f = 0,200 m^2

and which is quasi-static placed first on a cosine-like wave with a height of 7.92 m and then on a trochoidal wave of the same height.

Comparative calculations to verify the RLS-V1 program for cosine wave are given in Table 2.2.1, showing that the differences between manually determined values and program results are below 1%.

Table 2.2.1 – Comparative calculations to verify the RLS-V1 program for layout on cosine wave

Equilibrium parameters, additional efforts induced by cosine waves, deflection	**Manually determined values**		**Valori determinate de program**		**Differences in [%] between manual and program calculations**	
	On the crest	On the hollow	On the crest	On the hollow	On the crest	On the hollow
T_o [m]	4.878	4.878	4.878	4.878	0.000	0.000
ψ [rad]	0.000	0.000	0.000	0.000	0.000	0.000
Θ [rad]	0	0	0	0	0.000	0.000
$Q_{WV}(+)$ [kN]	12681	12681	12675	12675	0.047	0.047
$Q_{WV}(-)$ [kN]	-12681	-12681	-12675	-12675	0.047	0.047
M_{WV} [kNm]	403858	-403858	403449	-403448	0.101	0.102
Deflection [mm]	86.00	-86.00	85.15	-85.15	0.999	0.999

2.2.5 Comments and conclusions

The RLS-V1 program for calculating the general section efforts and the elastic line of the ship's hull, at static layout on still-water and at quasi-static layout on the cosine or trochoidal head wave, is a personal achievement and represent a useful tool for design and research activities to improve the construction safety of the ships. The program introduces several innovative elements as follows:

- determination of the elastic line of the hull and its influence on reduction of the bending sectional efforts by applying the transfer matrix method and by using finite macroelements;
- determining the reduction of bending sectional efforts due to the influence of the elastic line of the hull on the draughts along the ship length;
- determining of the sectional efforts in emergency situations.

2.3 Linear calculation program of the ship's oscillation parameters and of the additional sectional efforts in its hull induced by the head waves

2.3.1 Program object and destination

The program allows the linear determination of the ship's oscillation parameters as well as the additional sectional efforts in its hull induced by the head waves.

2.3.2 The classical method of linear calculation of the ship's oscillation parameters and of the additional sectional efforts in its hull induced by the head waves

The linear determination of the ship’s oscillation parameters and bending sectional efforts in the ship's hull in seagoing conditions, with regular head waves, is made principally in accordance with the method presented in [58], [59] and [60], developed and completed by the author in a specific way, to the level of details necessary for programming the calculation, taking into account only the heaving and the pitching (Fig. 2.3.1).

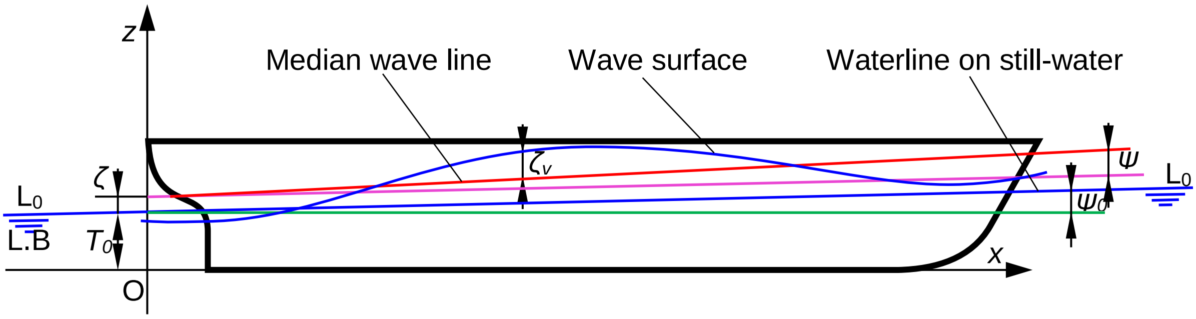


Fig. 2.3.1 – Dynamic layout of the ship on wave

To simplify the calculations, the following assumptions and considerations are made:

- the ship is considered a rigid body;
- the surface of the hull, described by points, is related to an orthogonal axis system (Figure 2.2.1). The points are arranged on theoretical couples located along the ship length and define the couple profile by draught and half-breadth;
- the sides are considered vertical in the variation zone of the draught;
- water is considered to be deep;
- the amplitude of the oscillations is considered to be small;
- the surface of the waves is considered as cosine-like shape in relation to the surface of the still-water, as described by the formula (see fig.2.3.2):

$$\zeta_V(x,t) = \frac{h_V}{2} \cdot \cos\left(\frac{2 \cdot \pi \cdot x}{\lambda_V} + \omega \cdot t\right) \qquad (2.3.1)$$

where:

λ_V – wave length

h_V – height wave

ω – pulsation of the wave;

t – time, variable;

$$\omega = \frac{2 \cdot \pi \cdot c}{\lambda_V} = \sqrt{\frac{2 \cdot \pi \cdot g}{\lambda_V}} \qquad (2.3.2)$$

c – wave speed;

If the real wave is regular, periodic and symmetrical to the crest, but has a different configuration than the cosine, the cosine-like equivalent wave is determined, the height of which is calculated from the requisite that the potential energy of the two waves is the same:

$$h_V = \sqrt{\frac{8 \cdot \int_0^L \zeta_{VR}(x) \cdot dx}{\lambda_V}} \qquad (2.3.3)$$

where:

$\zeta_{VR}(x)$ – the function that defines the real wave profile;

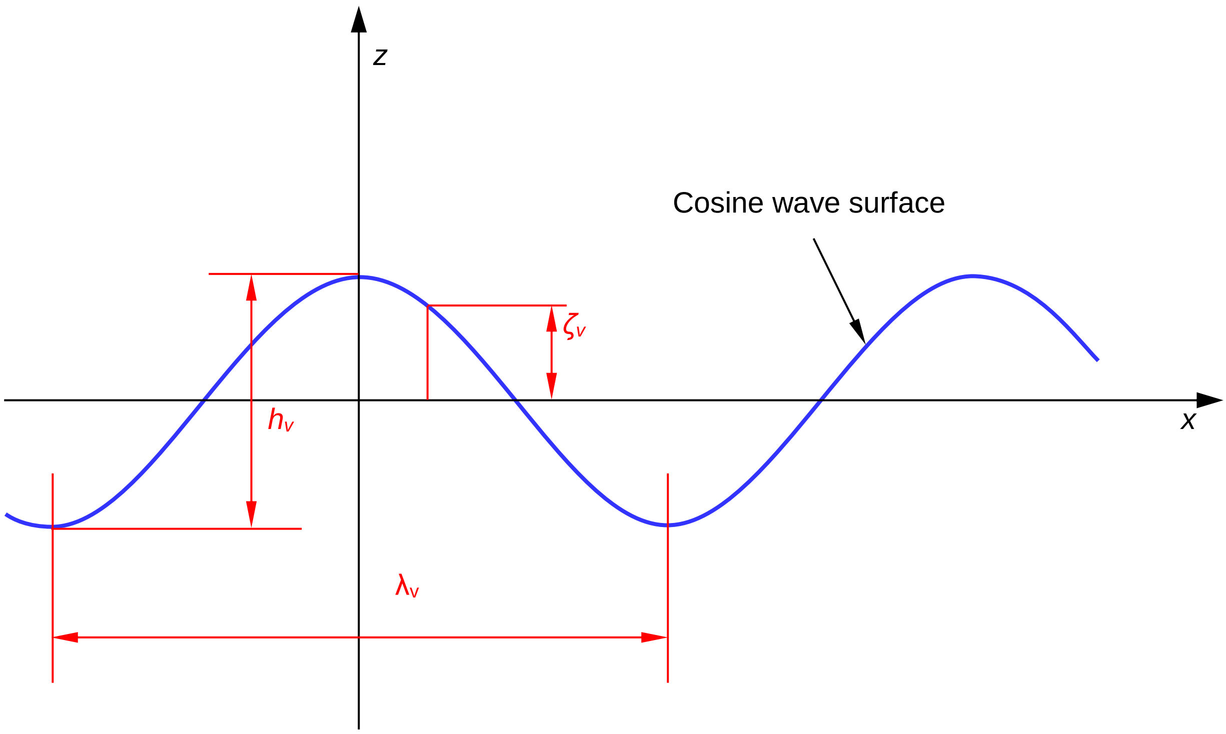


Fig. 2.3.2 – Cosine wave profile

- the total pressure in the wave is given by the formula taken from [61]:

$$p_t(x,z,t) = p_s(z) + p_w(x,z,t) \tag{2.3.4}$$

where:

$p_s(z)$ – represents the hydrostatic pressure in the wave;

$$p_s(z) = -\rho \cdot g \cdot z \tag{2.3.5}$$

$p_w(z)$ – represents the hydrodynamic pressure in the wave;

$$p_w(x,z,t) = \rho \cdot g \cdot e^{k\cdot[(z-\zeta_V(x,t)]} \cdot \zeta_V(x,t) \tag{2.3.6}$$

– the incoming waves are considered to be regular and approximate with a periodic function, the action of which is equivalent to the cosine waves:

$$\bar{\zeta}_V(x,t) = \frac{h_V}{2} \cdot f_1(x,t) \cdot \cos(\frac{2\cdot\pi\cdot x}{\lambda_V} + \omega_e \cdot t) = f_1(x,t) \cdot \zeta_V(x,t) \tag{2.3.7}$$

where:

$$\omega_e = \frac{2\cdot\pi\cdot(c+V)}{\lambda_V} = \omega + \frac{\omega^2}{g} \cdot V = \omega + k \cdot V \tag{2.3.8}$$

k – the wave number;

$$k = \frac{2\cdot\pi}{\lambda_V} \tag{2.3.9}$$

V – ship's speed;

$f_1(x,t)$ – the correction function which takes into account the effect of the additional pressure induced by the wave on the ship, known as the Smith effect.

In the specialized paper [63], the factor Smith f1 (x, t) is in the form of:

$$f_1(x,t) = e^{-kT_w(x,t)} \tag{2.3.10}$$

where:

$$T_w(x,t) = -\frac{1}{k}\cdot \ln\left[1 - \frac{2\cdot k}{b_m(x,t)}\cdot \int_{-T(x,t)}^{\varsigma_V(x,t)} y(x,z)\cdot e^{k[z-\varsigma_V(x,t)]}\cdot dz\right] \tag{2.3.11}$$

$b_m(x,t)$ – the mean breadth of the ship in the variation area of the draught *0-ζV(x,t)* from current section x;

- the vessel vertically oscillates around the equilibrium position in calm water, the size of which is denoted by $\zeta(t)$;
- the ship executes pitch oscillations around its center of gravity and its equilibrium position in calm water, the size of which is denoted by $\psi(t)$;
- the vertical displacement of a section x of the ship relative to the surface of the calm water due to the ship's oscillations is given by:

$$z(x,t) = \zeta(t) - x\cdot\psi(t) \tag{2.3.12}$$

- the vertical speed of a section *x* of the ship relative to the surface of the still water is given by the following relation:

$$\dot z(x,t) = \dot\zeta(t) - x\cdot\dot\psi(t) \tag{2.3.13}$$

- the vertical acceleration of a section *x* of the ship relative to the surface of the still water is given by the following relation:

$$\ddot z(x,t) = \ddot\zeta(t) - x\cdot\ddot\psi(t) \tag{2.3.14}$$

- the relative vertical displacement of a water particle relative to the surface of the ship's hull due to its oscillations is given by the following relation:

$$z_r(x,t) = -\zeta(t) + x\cdot\psi(t) + \overline{\zeta}_V(x,t) = -z(x,t) + \overline{\zeta}_V(x,t) \tag{2.3.15}$$

and the derivatives of this shift are given by the relations:

$$\frac{Dz_r(x,t)}{Dt} = -\dot\zeta(t) + x\cdot\dot\psi(t) - V\cdot\psi(t) + \frac{D\overline{\zeta}_V(x,t)}{Dt} \tag{2.3.16}$$

$$\frac{D^2 z_r(x,t)}{Dt^2} = -\ddot\zeta(t) + x\cdot\ddot\psi(t) - 2\cdot V\cdot\dot\psi(t) + \frac{D^2\overline{\zeta}_V(x,t)}{Dt^2} \tag{2.3.17}$$

$$\frac{D\overline{\zeta}_V(x,t)}{Dt} = -\frac{h_V}{2}\cdot f_1(x,t)\cdot\omega\cdot\sin\left(\frac{2\cdot\pi\cdot x}{\lambda_V} + \omega_e\cdot t\right) \tag{2.3.18}$$

$$\frac{D^2\overline{\zeta}_V(x,t)}{Dt^2} = -\frac{h_V}{2}\cdot f_1(x,t)\cdot\omega^2\cdot\cos\left(\frac{2\cdot\pi\cdot x}{\lambda_V} + \omega_e\cdot t\right) \tag{2.3.19}$$

- the draught measured from the surface of the still-water, in a current section for calculation *x*, is given by the relation:

$$T(x,t) = T_o - \zeta(t) + x\cdot[\psi_o + \psi(t)] \tag{2.3.20}$$

where:

T_o – the draught at the aft end of the ship in steady state on still water;

ψ_o – the trim angle of the vessel in steady state on still water;

$$T_o = T_{os} + \Delta T_V \tag{2.3.21}$$

T_{os} – draught at the aft end of the vessel in steady state when stationary on still water;

ΔT_V – the increase of the ship's draught when moving at the speed of V_C in knots, relative to the still water, due to the suction phenomenon, determined by the formula taken from [64]:

$$\Delta T_V = \frac{C_B \cdot K_S^{0.81} \cdot V_C^{2.08}}{20} \quad (2.3.22);$$

in canals or in test basins

$$\Delta T_V = \frac{C_B \cdot V_C^2}{100} \cdot (L/200)^{2/3} \qquad \text{the high seas} \quad (2.3.23);$$

$$K_S = \frac{S_N}{S_C} \tag{2.3.24}$$

where:

S_N – Immersion area of the ship's transverse section to the amidships;

S_C – Area of the transverse section of the channel;

V_C – Ship's speed in channel, expressed in knots.

Taking into account that the draught in still water is given by the formula:

$$T_S(x) = T_o + x \cdot \psi_o \tag{2.3.25}$$

the expression of the draught in the current x section becomes:

$$T(x,t) = T_S(x) - z(x,t) \tag{2.3.26}$$

The ship's oscillations on waves are defined by the two ζ and ψ parameters which can be determined according to the D'Alambert principle from the dynamic equilibrium equations under the action of vertical loads of inertia, damping, hydrostatic, hydrodynamic and weights.

The ship's hull to be is considered meshed in elements of transverse strips of dx length and by insulating such an element, on it acts the following loads:

– inertial loads of the distributed mass of the element, from the body structure and the cargoes:

$$q_i(x,t) = -m(x) \cdot \ddot{z}(x,t) \cdot dx \tag{2.3.27}$$

where:

m(x) – the distributed mass of the ship, from the body structure and the cargoes;

– the inertial loads of the distributed mass of elementary water added to the element, coupled with those of the hydrodynamic damping, according to "Ordinary Strip Theory (TFO)", Gerritsma and Beukelman (semiempiric) variant, published in [62], given by the formula:

$$q_a(x,t) = \left\{ \frac{D}{Dt}\left[M_{33}(x,t) \cdot \frac{Dz_r(x,t)}{Dt} \right] + N_{33}(x,t) \cdot \frac{Dz_r(x,t)}{Dt} \right\} \cdot dx \tag{2.3.28}$$

where:

$M_{33}(x,t)$ – the distributed mass of additional water in section x;

$N_{33}(x,t)$ – the hydrodynamic damping coefficient in section x.

Parameters $M_{33}(x,t)$ and $N_{33}(x,t)$ are determined by model tests or by calculation methodologies and indicated in the literature, in tabular or graphic form. In the case of this paper, these parameters are determined based on the data from the paper [59], using the formulas below:

$$M_{33}(x,t) = \pi \cdot \rho \cdot b^2(x,t)/8 \cdot c_{33}[\omega_e^2 \cdot T_T(x,t)/g,\ 2 \cdot T_T(x,t)/b(x,t), \beta(x,t)] \qquad (2.3.29)$$

$$N_{33}(x,t) = \omega_e \cdot \rho \cdot b^2(x,t)/4 \cdot \lambda_{33}[\omega_e^2 \cdot T_T(x,t)/g,\ 2 \cdot T_T(x,t)/b(x,t), \beta(x,t)] \qquad (2.3.30)$$

where:

$$T_T(x,t) = T(x,t) + \zeta_V(x,t) \qquad (2.3.31)$$

$b(x,T_T)$ – the width of the ship at the draught $T_T(x,t)$ in the current section x;

$\beta(x,T_T)$ – the coefficient of the section's area x at the draught $T_T(x,t)$;

the coefficients c_{33} and λ_{33} are determined from the diagrams presented in [59].

– the same inertial loads of the distributed mass of additional water of the element, coupled with the hydrodynamic amortization, according to "Modified Strip Theory - (TFM)" published by Tasai in 1969 [65], are given by the formula:

$$q_a(x,t) = \frac{D}{Dt}\left\{\left[M_{33}(x,t) - \frac{i}{\omega_e} \cdot N_{33}(x,t)\right] \cdot \frac{Dz_r(x,t)}{Dt}\right\} \cdot dx \qquad (2.3.32)$$

which by derivation becomes:

$$q_a(x,t) = \left[M_{33}(x,t) + \frac{V}{\omega_e^2} \cdot \frac{dN_{33}(x,t)}{dx}\right] \cdot \frac{D^2 z_r(x,t)}{Dt^2} \cdot dx + \qquad (2.3.33)$$
$$+ \left[N_{33}(x,t) - V \cdot \frac{dM_{33}(x,t)}{dx}\right] \cdot \frac{Dz_r(x,t)}{Dt} \cdot dx$$

It is noted that the modified theory differs from the ordinary one by adding the term:

$$\frac{V}{\omega_e^2} \cdot \frac{dN_{33}(x,t)}{dx} \cdot \frac{D^2 z_r(x,t)}{Dt^2} \qquad (2.3.34)$$

from relation (2.3.33).

– hydrostatic loads:

$$q_h(x,t) = 2 \cdot \rho \cdot g \cdot y(x,T) \cdot [T_S(x) - z(x,t) + \bar{\zeta}_V(x,t)] \cdot \beta(x,T) \cdot dx \qquad (2.3.35)$$

where:

$y(x,T)$ – half-breadth of the hull in section x to the waterline in steady state on still water. Shell plating in the draught variation zone during vertical oscillations of the ship is considered vertical;

$\beta(x,T)$ – the coefficient of the area of the section x at the waterline in steady state on still water;

The expression of hydrostatic loads can also be written as:

$$q_h(x,t) = 2 \cdot \rho \cdot g \cdot y(x,T_S) \cdot T_S(x) \cdot \beta(x,T_S) \cdot dx -$$
$$-2 \cdot \rho \cdot g \cdot y(x,T_S) \cdot [z(x,t) - \bar{\zeta}_V(x,t)] \cdot dx \qquad (2.3.36)$$

It is noted that the first term represents hydrostatic thrust in still water:

$$q_{hS}(x) = 2 \cdot \rho \cdot g \cdot y(x,T_S) \cdot T_S(x) \cdot \beta(x,T_S) \cdot dx \qquad (2.3.37)$$

and the second term represents additional hydrostatic thrust due to ship oscillation and wave:

$$q_{hW}(x,t) = -2\cdot\rho\cdot g\cdot y(x,T_S)\cdot[z(x,t)-\overline{\zeta}_V(x,t)]\cdot dx \tag{2.3.38}$$

- the weight of the distributed mass of the strip element, derived from the hull structure and the cargoes:

$$q_g(x) = g\cdot m(x)\cdot dx \tag{2.3.39}$$

- the shear forces at the ends of the element, which represents the connecting forces: Q_T and Q_T+dQ_T;
- the bending moments at the ends of the element, which represents the connecting forces: M and $M+dM$.

By writing the dynamic equilibrium equation of loads in the vertical direction for a transverse element, it is determined the elementary increase of the shear force:

$$dQ_T(x,t) = q_i(x,t)+q_a(x,t)+q_{hS}(x)+q_{hW}(x,t)-q_g(x) \tag{2.3.40}$$

By integrating this relation along the ship length, we obtain the expression of the total shear force resulting from the loads in still water and from the additional loads due to the ship and wave oscillations:

$$Q_T(x,t) = \int_0^x [q_i(\xi,t)+q_a(\xi,t)+q_{hS}(\xi)+q_{hW}(\xi,t)-q_g(\xi)]\cdot d\xi \tag{2.3.41}$$

By regrouping the terms, the expression of the total shear force can also be written as follow:

$$Q_T(x,t) = \int_0^x [q_{hS}(\xi)-q_g(\xi)]\cdot d\xi + \int_0^x [q_i(\xi,t)+q_a(\xi,t)+q_{hW}(\xi,t)]\cdot d\xi \tag{2.3.42}$$

The first term of this expression is the shear force in still water:

$$Q_S(x) = \int_0^x [q_{hS}(\xi)-q_g(\xi)]\cdot d\xi \tag{2.3.43}$$

and the second term of this expression is the additional shear force due to the ship and wave oscillations:

$$Q_W(x,t) = \int_0^x [q_i(\xi,t)+q_a(\xi,t)+q_{hW}(\xi,t)]\cdot d\xi \tag{2.3.44}$$

The total bending moment along the ship length is obtained by integrating the relation (2.3.41):

$$M_T(x,t) = \int_0^x Q_T(\xi,t)\cdot d\xi \tag{2.3.45}$$

and taking into account the relations (2.3.42), (2.3.43) and (2.3.44), it follows:

$$M_T(x,t) = \int_0^x Q_S(\xi)\cdot d\xi + \int_0^x Q_W(\xi,t)\cdot d\xi \tag{2.3.46}$$

where the first term of this expression is the bending moment in still water:

$$M_S(x) = \int_0^x Q_S(\xi)\cdot d\xi \tag{2.3.47}$$

and the second term represents the additional bending moment due to the ship's and waves oscillations:

$$M_W(x,t) = \int_0^x Q_W(\xi,t) \cdot d\xi \tag{2.3.48}$$

By integrating the expressions (2.3.47) and (2.3.48) by part, is obtained:

$$M_S(x) = x \cdot Q_S(x) - \int_0^x \xi \cdot [q_{hS}(\xi) - q_g(\xi)] \cdot d\xi \tag{2.3.49}$$

$$M_W(x,t) = x \cdot Q_W(x,t) - \int_0^x \xi \cdot [q_i(\xi,t) + q_a(\xi,t) + q_{hW}(\xi,t)] \cdot d\xi \tag{2.3.50}$$

Taking into account that at the forward extremity, the total shear force, the shear force in still water and the additional shear force have the zero value, the relation (2.3.44) becomes:

$$\int_0^L [q_i(\xi,t) + q_a(\xi,t) + q_{hW}(\xi,t)] \cdot d\xi = 0 \tag{2.3.51}$$

Taking into account that at the forward extremity, the total bending moment, the bending moment in still water and the additional bending moment have the zero value, as well as the fact that in the same section the shear force in still water and the additional shear force have the zero value, the relation (2.3.50) becomes:

$$\int_0^L \xi \cdot [q_i(\xi,t) + q_a(\xi,t) + q_{hW}(\xi,t)] \cdot d\xi = 0 \tag{2.3.52}$$

Starting from the relations (2.3.51) and (2.3.52) and taking into account formulas (2.3.12) - (2.3.19), (2.3.27), (2.3.28) and (2.3.33), by replacements and processing, a system of 2 differential equations is obtained:

$$\begin{cases} A_{\zeta\zeta}(t) \cdot \ddot{\zeta} + B_{\zeta\zeta}(t) \cdot \dot{\zeta} + C_{\zeta\zeta}(t) \cdot \zeta + A_{\zeta\psi}(t) \cdot \ddot{\psi} + B_{\zeta\psi}(t) \cdot \dot{\psi} + C_{\zeta\psi}(t) \cdot \psi = F_V(t) \\ A_{\psi\zeta}(t) \cdot \ddot{\zeta} + B_{\psi\zeta}(t) \cdot \dot{\zeta} + C_{\psi\zeta}(t) \cdot \zeta + A_{\psi\psi}(t) \cdot \ddot{\psi} + B_{\psi\psi}(t) \cdot \dot{\psi} + C_{\psi\psi}(t) \cdot \psi = M_V(t) \end{cases} \tag{2.3.53}$$

where:

$$A_{\zeta\zeta}(t) = \int_0^L [m(x) + M_{33}(x,t) + \frac{V}{\omega_e^2} \cdot \frac{dN_{33}(x,t)}{dx}] \cdot dx \tag{2.3.54}$$

$$B_{\zeta\zeta}(t) = \int_0^L [N_{33}(x,t) - V \cdot \frac{dM_{33}(x,t)}{dx}] \cdot dx \tag{2.3.55}$$

$$C_{\zeta\zeta}(t) = 2 \cdot \rho \cdot g \cdot \int_0^L y(x,t) \cdot dx \tag{2.3.56}$$

$$A_{\zeta\psi}(t) = -\int_0^L x \cdot [m(x) + M_{33}(x,t) + \frac{V}{\omega_e^2} \cdot \frac{dN_{33}(x,t)}{dx}] \cdot dx \tag{2.3.57}$$

$$B_{\zeta\psi}(t) = -\int_0^L \{-2 \cdot V \cdot [M_{33}(x,t) + \frac{V}{\omega_e^2} \cdot \frac{dN_{33}(x,t)}{dx}] + \tag{2.3.58}$$

$$+ x \cdot [N_{33}(x) - V \cdot \frac{dM_{33}(x,t)}{dx}]\}dx$$

$$C_{\zeta\psi}(t) = -\int_0^L \{2\cdot\rho\cdot g\cdot x\cdot y(x,t) - V\cdot[N_{33}(x,t) - V\cdot\frac{dM_{33}(x,t)}{dx}]\}\cdot dx \tag{2.3.59}$$

$$F_V(t) = -\omega^2\cdot\frac{h_V}{2}\cdot\int_0^L f_1(x,t)\cdot[M_{33}(x,t) + \frac{V}{\omega_e^2}\cdot\frac{dN_{33}(x,t)}{dx}]\cdot\cos(\frac{2\cdot\pi\cdot x}{\lambda_V} + \omega_e\cdot t)\cdot dx -$$

$$-\omega\cdot\frac{h_V}{2}\cdot\int_0^L f_1(x,t)\cdot[N_{33}(x,t) - V\cdot\frac{dM_{33}(x,t)}{dx}]\cdot\sin(\frac{2\cdot\pi\cdot x}{\lambda_V} + \omega_e\cdot t)\cdot dx +$$

$$+\rho\cdot g\cdot h_V\cdot\int_0^L f_1(x,t)\cdot y(x)\cdot\cos(\frac{2\cdot\pi\cdot x}{\lambda_V} + \omega_e\cdot t)\cdot dx =$$

$$= F_{VC}(t)\cdot\cos(\omega_e t) + F_{VS}(t)\cdot\sin(\omega_e t) \tag{2.3.60}$$

in which it was noted:

$$F_{VC}(t) = -\omega^2\cdot\frac{h_V}{2}\cdot\int_0^L f_1(x,t)\cdot[M_{33}(x,t) + \frac{V}{\omega_e^2}\cdot\frac{dN_{33}(x,t)}{dx}]\cdot\cos(\frac{2\cdot\pi\cdot x}{\lambda_V})\cdot dx -$$

$$-\omega\cdot\frac{h_V}{2}\cdot\int_0^L f_1(x,t)\cdot[N_{33}(x,t) - V\cdot\frac{dM_{33}(x,t)}{dx}]\cdot\sin(\frac{2\cdot\pi\cdot x}{\lambda_V})\cdot dx +$$

$$+\rho\cdot g\cdot h_V\cdot\int_0^L f_1(x,t)\cdot y(x,t)\cdot\cos(\frac{2\cdot\pi\cdot x}{\lambda_V})\cdot dx \tag{2.3.61}$$

$$F_{VS}(t) = \omega^2\cdot\frac{h_V}{2}\cdot\int_0^L f_1(x,t)\cdot[M_{33}(x,t) + \frac{V}{\omega_e^2}\cdot\frac{dN_{33}(x,t)}{dx}]\cdot\sin(\frac{2\cdot\pi\cdot x}{\lambda_V})\cdot dx -$$

$$-\omega\cdot\frac{h_V}{2}\cdot\int_0^L f_1(x,t)\cdot[N_{33}(x,t) - V\cdot\frac{dM_{33}(x,t)}{dx}]\cdot\cos(\frac{2\cdot\pi\cdot x}{\lambda_V})\cdot dx +$$

$$-\rho\cdot g\cdot h_V\cdot\int_0^L f_1(x,t)\cdot y(x,t)\cdot\sin(\frac{2\cdot\pi\cdot x}{\lambda_V})\cdot dx \tag{2.3.62}$$

$$A_{\psi\zeta}(t) = \int_0^L x\cdot[m(x) + M_{33}(x,t) + \frac{V}{\omega_e^2}\cdot\frac{dN_{33}(x,t)}{dx}]\cdot dx \tag{2.3.63}$$

$$B_{\psi\zeta}(t) = \int_0^L x\cdot[N_{33}(x,t) - V\cdot\frac{dM_{33}(x,t)}{dx}]\cdot dx \tag{2.3.64}$$

$$C_{\psi\zeta}(t) = 2\cdot\rho\cdot g\cdot\int_0^L x\cdot y(x,t)\cdot dx \tag{2.3.65}$$

$$A_{\psi\psi}(t) = -\int_0^L x^2\cdot[m(x) + M_{33}(x,t) + \frac{V}{\omega_e^2}\cdot\frac{dN_{33}(x,t)}{dx}]\cdot dx \tag{2.3.66}$$

$$B_{\psi\psi}(t) = -\int_0^L x \cdot \{-2 \cdot V \cdot [M_{33}(x,t) + \frac{V}{\omega_e^2} \cdot \frac{dN_{33}(x,t)}{dx}] + \\ + x \cdot [N_{33}(x,t) - V \cdot \frac{dM_{33}(x,t)}{dx}]\} \cdot dx \tag{2.3.67}$$

$$C_{\psi\psi}(t) = -\int_0^L \{2 \cdot \rho \cdot g \cdot x^2 \cdot y(x,t) - V \cdot x \cdot [N_{33}(x,t) - V \cdot \frac{dM_{33}(x,t)}{dx}]\} \cdot dx \tag{2.3.68}$$

$$M_V(t) = -\omega^2 \cdot \frac{h_V}{2} \cdot \int_0^L f_1(x,t) \cdot x \cdot [M_{33}(x,t) + \frac{V}{\omega_e^2} \cdot \frac{dN_{33}(x,t)}{dx}] \cdot \cos(\frac{2 \cdot \pi \cdot x}{\lambda_V} + \omega_e \cdot t) \cdot dx - \\ -\omega \cdot \frac{h_V}{2} \cdot \int_0^L f_1(x,t) \cdot x \cdot [N_{33}(x,t) - V \cdot \frac{dM_{33}(x,t)}{dx}] \cdot \sin(\frac{2 \cdot \pi \cdot x}{\lambda_V} + \omega_e \cdot t) \cdot dx + \\ + \rho \cdot g \cdot h_V \cdot \int_0^L f_1(x,t) \cdot x \cdot y(x,t) \cdot \cos(\frac{2 \cdot \pi \cdot x}{\lambda_V} + \omega_e \cdot t) \cdot dx = \\ = M_{VC}(t) \cdot \cos(\omega_e t) + M_{VS}(t) \cdot \sin(\omega_e t) \tag{2.3.69}$$

in which it was noted:

$$M_{VC}(t) = -\omega^2 \cdot \frac{h_V}{2} \cdot \int_0^L f_1(x,t) \cdot x \cdot [M_{33}(x,t) + \frac{V}{\omega_e^2} \cdot \frac{dN_{33}(x,t)}{dx}] \cdot \cos(\frac{2 \cdot \pi \cdot x}{\lambda_V}) \cdot dx - \\ -\omega \cdot \frac{h_V}{2} \cdot \int_0^L f_1(x,t) \cdot x \cdot [N_{33}(x,t) - V \cdot \frac{dM_{33}(x,t)}{dx}] \cdot \sin(\frac{2 \cdot \pi \cdot x}{\lambda_V}) \cdot dx + \\ + \rho \cdot g \cdot h_V \cdot \int_0^L f_1(x,t) \cdot x \cdot y(x,t) \cdot \cos(\frac{2 \cdot \pi \cdot x}{\lambda_V}) \cdot dx \tag{2.3.70}$$

$$M_{VS}(t) = \omega^2 \cdot \frac{h_V}{2} \cdot \int_0^L f_1(x,t) \cdot x \cdot [M_{33}(x,t) + \frac{V}{\omega_e^2} \cdot \frac{dN_{33}(x,t)}{dx}] \cdot \sin(\frac{2 \cdot \pi \cdot x}{\lambda_V}) \cdot dx - \\ -\omega \cdot \frac{h_V}{2} \cdot \int_0^L f_1(x,t) \cdot x \cdot [N_{33}(x,t) - V \cdot \frac{dM_{33}(x,t)}{dx}] \cdot \cos(\frac{2 \cdot \pi \cdot x}{\lambda_V}) \cdot dx - \\ - \rho \cdot g \cdot h_V \cdot \int_0^L f_1(x,t) \cdot x \cdot y(x,t) \cdot \sin(\frac{2 \cdot \pi \cdot x}{\lambda_V}) \cdot dx \tag{2.3.71}$$

Taking into account the simplifying assumptions made by the fact that the behavior of the vessel in the field of small oscillations is studied, when its influence on the parameters M_{33} and N_{33} can be neglected, the system of second order differential equations (2.3.53) is considered to be linear and its solution is stabilized and has the form of external excitation loads $F_V(t)$ and $M_V(t)$:

$$\zeta(t) = \zeta_1 \cdot \cos(\omega_e \cdot t) + \zeta_2 \cdot \sin(\omega_e \cdot t) = \zeta_a \cdot \cos(\omega_e \cdot t + \varepsilon_\zeta) \tag{2.3.72}$$

$$\psi(t) = \psi_1 \cdot \cos(\omega_e \cdot t) + \psi_2 \cdot \sin(\omega_e \cdot t) = \psi_a \cdot \cos(\omega_e \cdot t + \varepsilon_\psi) \quad (2.3.73)$$

Between the above parameters, there are the relationships:

$$\zeta_a = \sqrt{\zeta_1^2 + \zeta_2^2} \quad (2.3.74)$$

$$\psi_a = \sqrt{\psi_1^2 + \psi_2^2} \quad (2.3.75)$$

$$\varepsilon_\zeta = -arctg \frac{\zeta_2}{\zeta_1} \quad (2.3.76)$$

$$\varepsilon_\psi = -arctg \frac{\psi_2}{\psi_1} \quad (2.3.77)$$

By replacing the solutions (2.3.72) and (2.3.73) in the system (2.3.53) and deriving and grouping by cos and sin, we obtain the system of two equations in which the unknowns are $\zeta_1, \zeta_2, \psi_1, \psi_2$:

$$\begin{cases} \{[C_{\zeta\zeta}(t) - \omega_e^2 A_{\zeta\zeta}(t)] \cdot \zeta_1 + \omega_e \cdot B_{\zeta\zeta}(t) \cdot \zeta_2 + [C_{\zeta\psi}(t) - \omega_e^2 A_{\zeta\psi}(t)] \cdot \psi_1 + \omega_e \cdot B_{\zeta\psi}(t) \cdot \psi_2\} \cdot \cos(\omega_e \cdot t) + \\ \{-\omega_e \cdot B_{\zeta\zeta}(t) \cdot \zeta_1 + [C_{\zeta\zeta}(t) - \omega_e^2 A_{\zeta\zeta}(t)] \cdot \zeta_2 - \omega_e \cdot B_{\zeta\psi}(t) \cdot \psi_1 + [C_{\zeta\psi}(t) - \omega_e^2 A_{\zeta\psi}(t)] \cdot \psi_2\} \cdot \sin(\omega_e \cdot t) = \\ = F_{VC}(t) \cdot \cos(\omega_e \cdot t) + F_{VS}(t) \cdot \sin(\omega_e \cdot t) \\ \\ \{[C_{\psi\zeta}(t) - \omega_e^2 A_{\psi\zeta}(t)] \cdot \zeta_1 + \omega_e \cdot B_{\psi\zeta}(t) \cdot \zeta_2 + [C_{\psi\psi}(t) - \omega_e^2 A_{\psi\psi}(t)] \cdot \psi_1 + \omega_e \cdot B_{\psi\psi}(t) \cdot \psi_2] \cdot \cos(\omega_e \cdot t) + \\ \{-\omega_e \cdot B_{\psi\zeta}(t) \cdot \zeta_1 + [C_{\psi\zeta}(t) - \omega_e^2 A_{\psi\zeta}(t)] \cdot \zeta_2 - \omega_e \cdot B_{\psi\psi}(t) \cdot \psi_1 + [C_{\psi\psi}(t) - \omega_e^2 A_{\psi\psi}(t)] \cdot \psi_2]\} \sin(\omega_e \cdot t) = \\ = M_{VC}(t) \cdot \cos(\omega_e \cdot t) + M_{VS}(t) \cdot \sin(\omega_e \cdot t) \end{cases} \quad (2.3.78)$$

Since there are only two equations of equilibrium, the 4 unknowns $\zeta_1, \zeta_2, \psi_1, \psi_2$ are determined by equilibration of the ship for the moments:

$$t_1 = 0$$
$$t_2 = \frac{\pi}{2 \cdot \omega_e} \quad (2.3.79)$$

resulting a system of 4 equations that can be written in matrix form:

$$[A] \cdot \{X\} = \{F\} \quad (2.3.80)$$

where:

$$A = \begin{bmatrix} C_{\zeta\zeta}(t_1) - \omega_e^2 A_{\zeta\zeta}(t_1) & \omega_e \cdot B_{\zeta\zeta}(t_1) & C_{\zeta\psi}(t_1) - \omega_e^2 A_{\zeta\psi}(t_1) & \omega_e \cdot B_{\zeta\psi}(t_1) \\ -\omega_e \cdot B_{\zeta\zeta}(t_2) & C_{\zeta\zeta}(t_2) - \omega_e^2 A_{\zeta\zeta}(t_2) & -\omega_e \cdot B_{\zeta\psi}(t_2) & C_{\zeta\psi}(t_2) - \omega_e^2 A_{\zeta\psi}(t_2) \\ C_{\psi\zeta}(t_1) - \omega_e^2 A_{\psi\zeta}(t_1) & \omega_e \cdot B_{\psi\zeta}(t_1) & C_{\psi\psi}(t_1) - \omega_e^2 A_{\psi\psi}(t_1) & \omega_e \cdot B_{\psi\psi}(t_1) \\ -\omega_e \cdot B_{\psi\zeta}(t_2) & C_{\psi\zeta}(t_2) - \omega_e^2 A_{\psi\zeta}(t_2) & -\omega_e \cdot B_{\psi\psi}(t_2) & C_{\psi\psi}(t_2) - \omega_e^2 A_{\psi\psi}(t_2) \end{bmatrix} \quad (2.3.81)$$

$$X = \{\zeta_1 \quad \zeta_2 \quad \psi_1 \quad \psi_2\}^T \quad (2.3.82)$$

$$F = \{F_{VC}(t_1) \quad F_{VS}(t_2) \quad M_{VC}(t_1) \quad M_{VS}(t_2)\}^T \quad (2.3.83)$$

By solving the system (2.3.80), the functions $\zeta(t)$ and $\psi(t)$ that characterize the ship's oscillations on the waves are determined, based on which are calculated the additional shear force and the additional bending moment when the ship moves on waves, starting from the relations (2.3.44) and (2.3.50).

2.3.3 Description of the RLD-V1 program

Based on the calculation method presented in 2.3.2 has been developed the RLD-V1 program for linear calculation of the ship's oscillation parameters and of the additional sectional efforts in its hull induced by the head waves, considering linear damping, whose code being written in the Visual-FORTRAN language that can be run on 32 or 64-bit computers running Windows XP operating system or a later version.

2.3.4 Verification of the calculation method and of the RLD-V1 program

The verification of the calculation method presented in 2.3.2 and of the RLD-V1 was performed first of all by comparing the results of the calculations provided by this program with the measurements made in 1992 on the Wigley III experimental model, by the Ship Hydrodinamics Laboratory of the University of Technology in Delft, in cooperation with the Department of Naval Architecture and Marine Engineering of the University of Michigan and the Hydrodynamics Committee of the Naval Architecture and Marine Engineering Society, presented in Works [66] and [67].

This model, shown in Fig. 2.3.3, has the following main features:

Length, L …………………….……..…………..……	3,0000 m
Breadth, B ……………………..…………………..….....	0,3000 m
Draught, T ……………………..………….........…..	0,1875 m
Trim, ψ_o ………………….......................……...…	0,0000 grd
Distance between couples, δ ……………….....…..	0,1500
Displacement, Δ ……………..…………………….....	0,0780 t
Block coefficient , C_B ……………………………........	0,4530
Amidships coeficient, C_T …………………………......	0,6667
Vertical coordinate of the center of gravity, KG	0,1700 m
Radius of inertia for pitch, R_Y ……………………......	0,7500 m

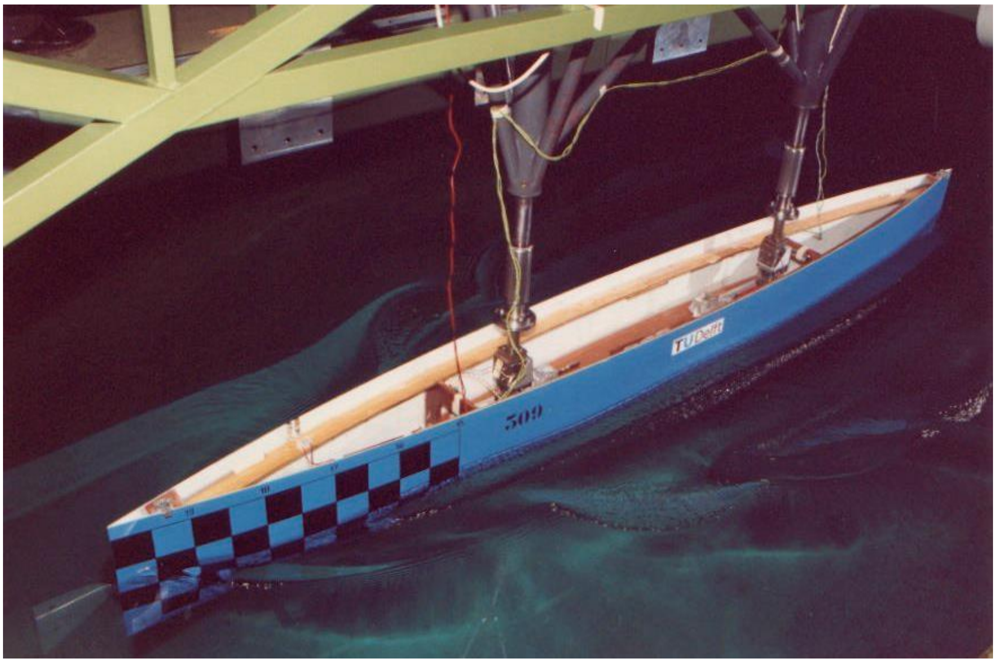


Fig. 2.3.3 – Wigley III model during the tests [67]

The comparison of the results of the calculations with the measurements made on the Wigley III experimental model was carried out at different navigation regimes characterized by F_n Froude number, defined by the formula:

$$F_n = \frac{V}{\sqrt{g \cdot L}} \tag{2.3.84}$$

using the dimensionless transfer functions for the vertical and pitch oscillations amplitudes defined by the relations:

$$\zeta_a^* = \frac{2 \cdot \zeta_a}{h_v} \tag{2.3.85}$$

$$\psi_a^* = \frac{\psi_a \cdot L}{\pi \cdot h_v} \tag{2.3.86}$$

as a dynamic response to the head waves action, characterized by the relative dimensionless pulsation, defined by the formula:

$$\omega^* = \omega \cdot \sqrt{\frac{L}{g}} \tag{2.3.87}$$

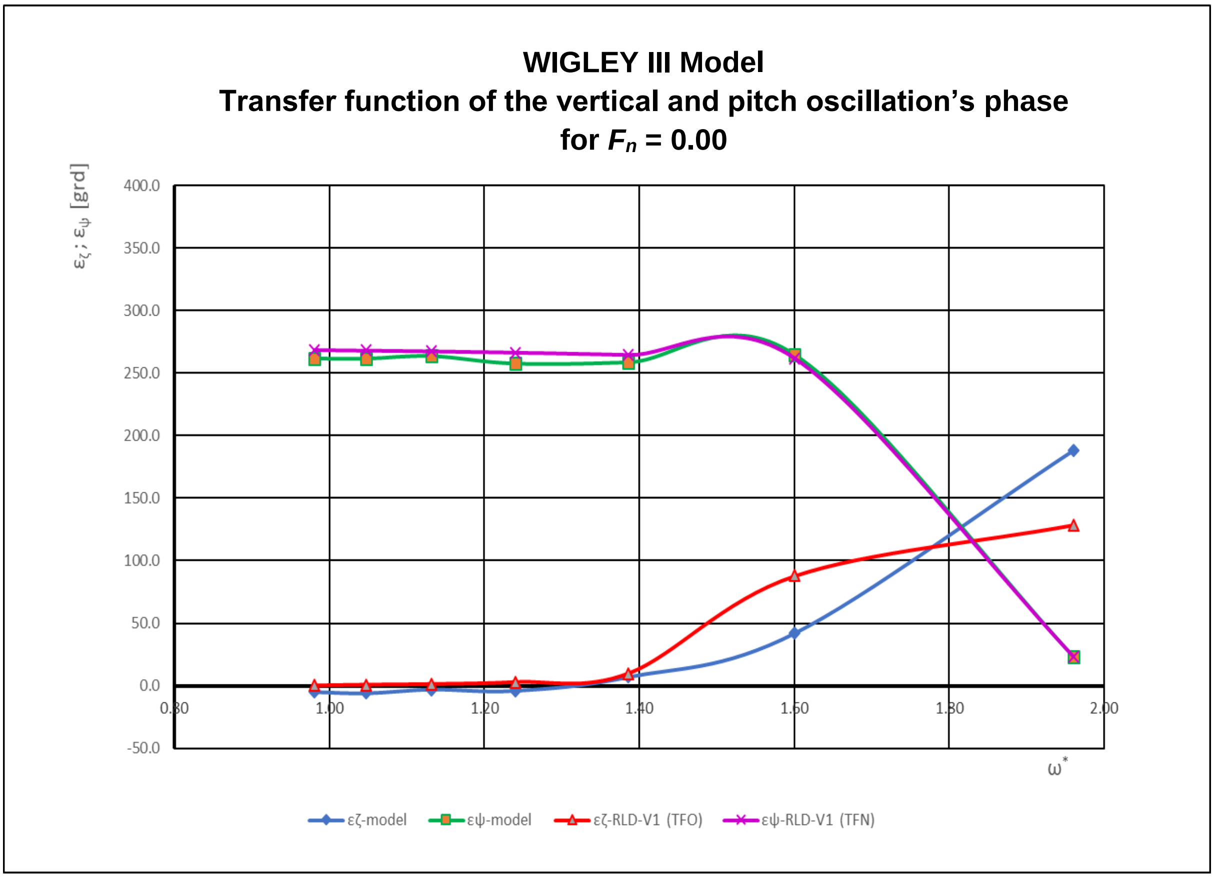
as well as the phases of these oscillations relative to the crest of the wave considered to be at the ship’s center of gravity.

Figures 2.3.4 - 2.3.6 graphically show the results of the calculations according to the presented method and the measurements made on the model, at corresponding speeds in which F_n had the values 0 and 0.20, on regular waves of 0.04 m height and the relative length defined as the ratio:

$$\lambda_V^* = \frac{\lambda_V}{L} \tag{2.3.88}$$

having the values: 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00.

In the graphs, the TFO indicates the use of the "Ordinary Strip Theory", and the TFM indicates the use of "Modified Strip Theory".



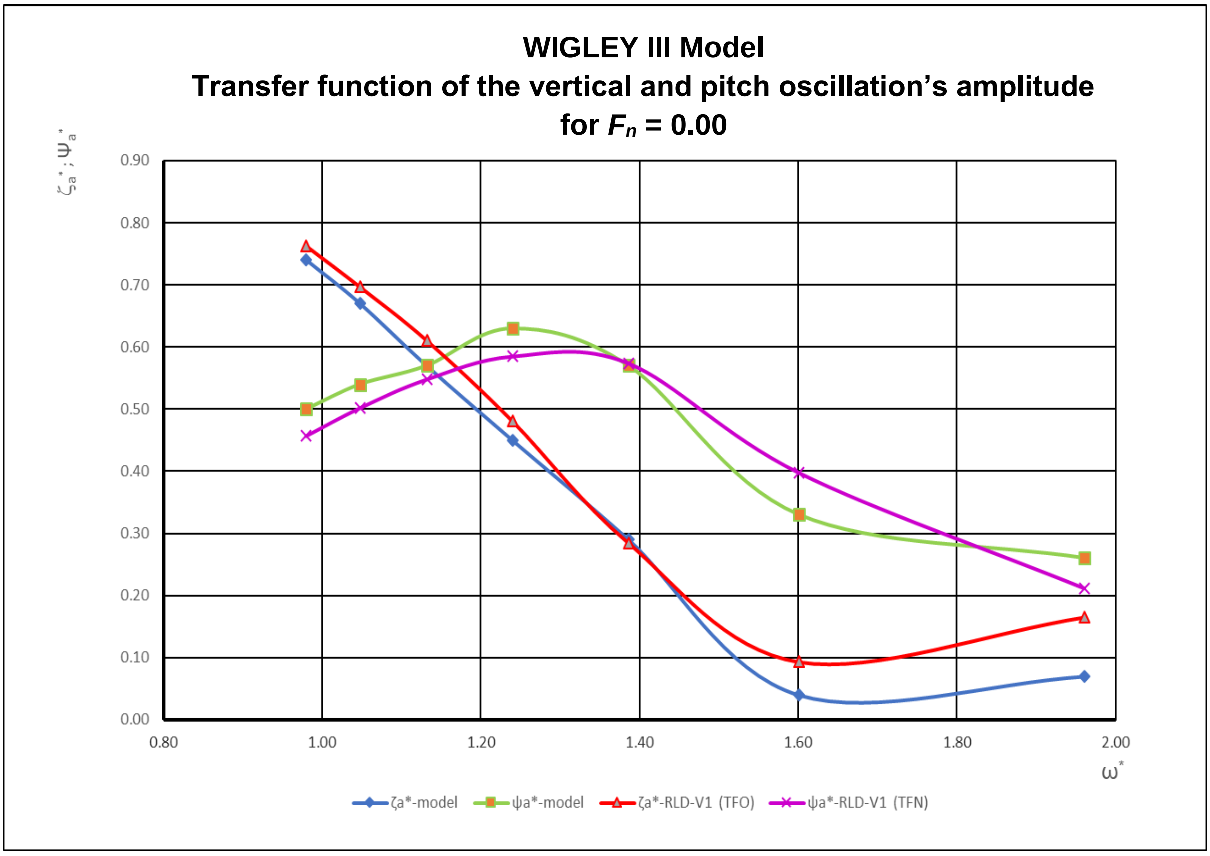
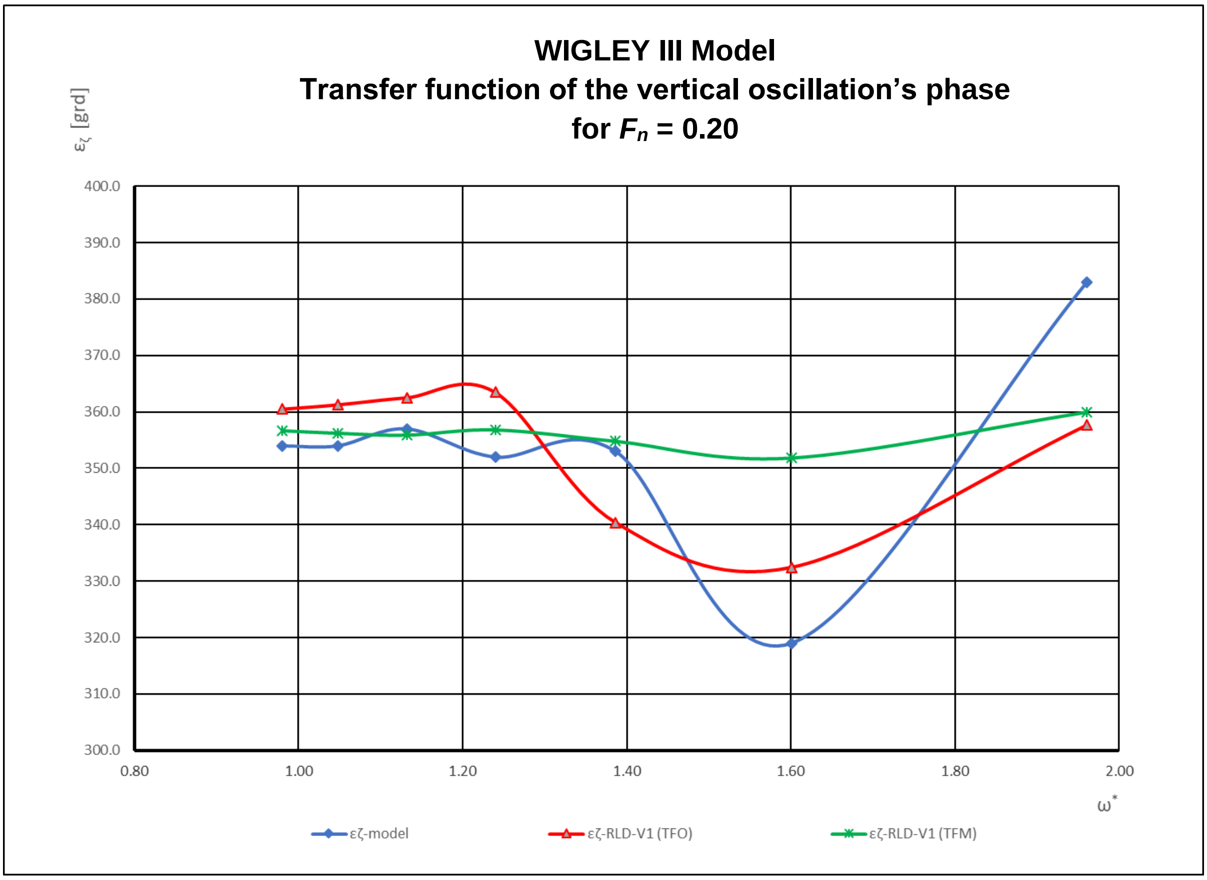


Fig. 2.3.4 – Dynamic response for the vertical and pitch oscillations at F_n = 0.00. The measurements on the Wigley III model are taken from [66]



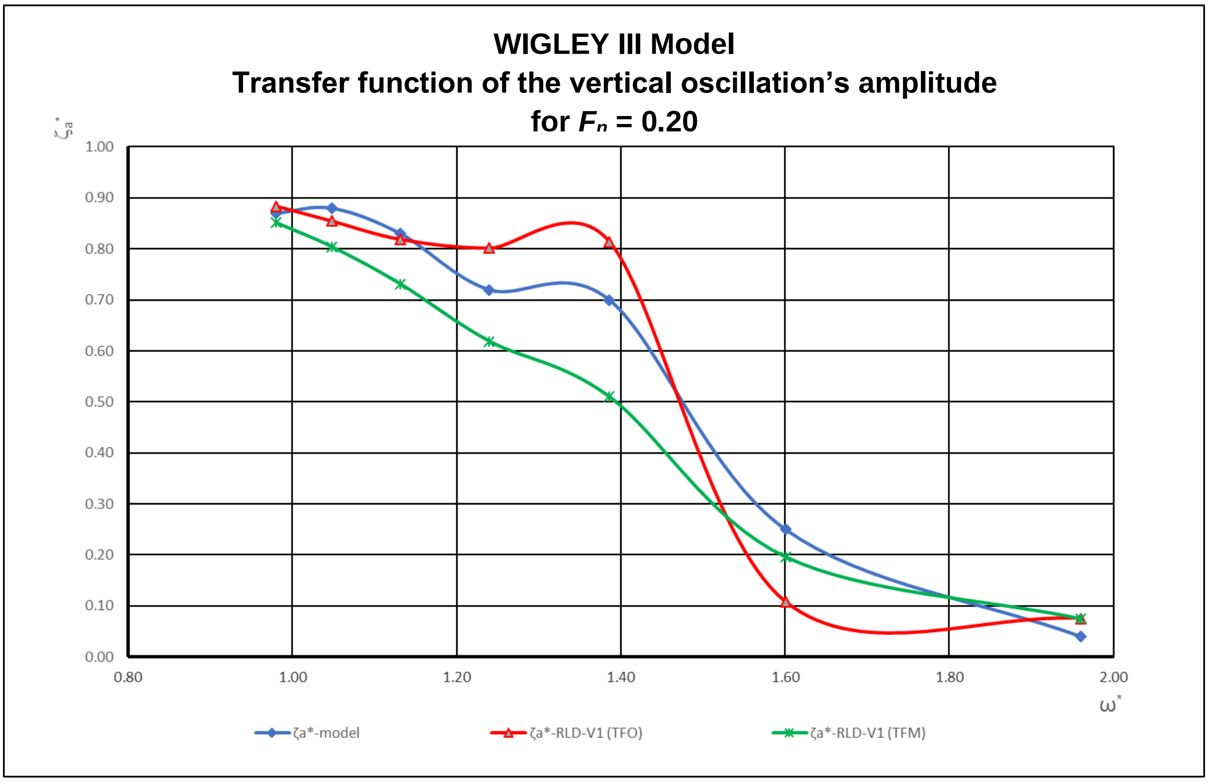
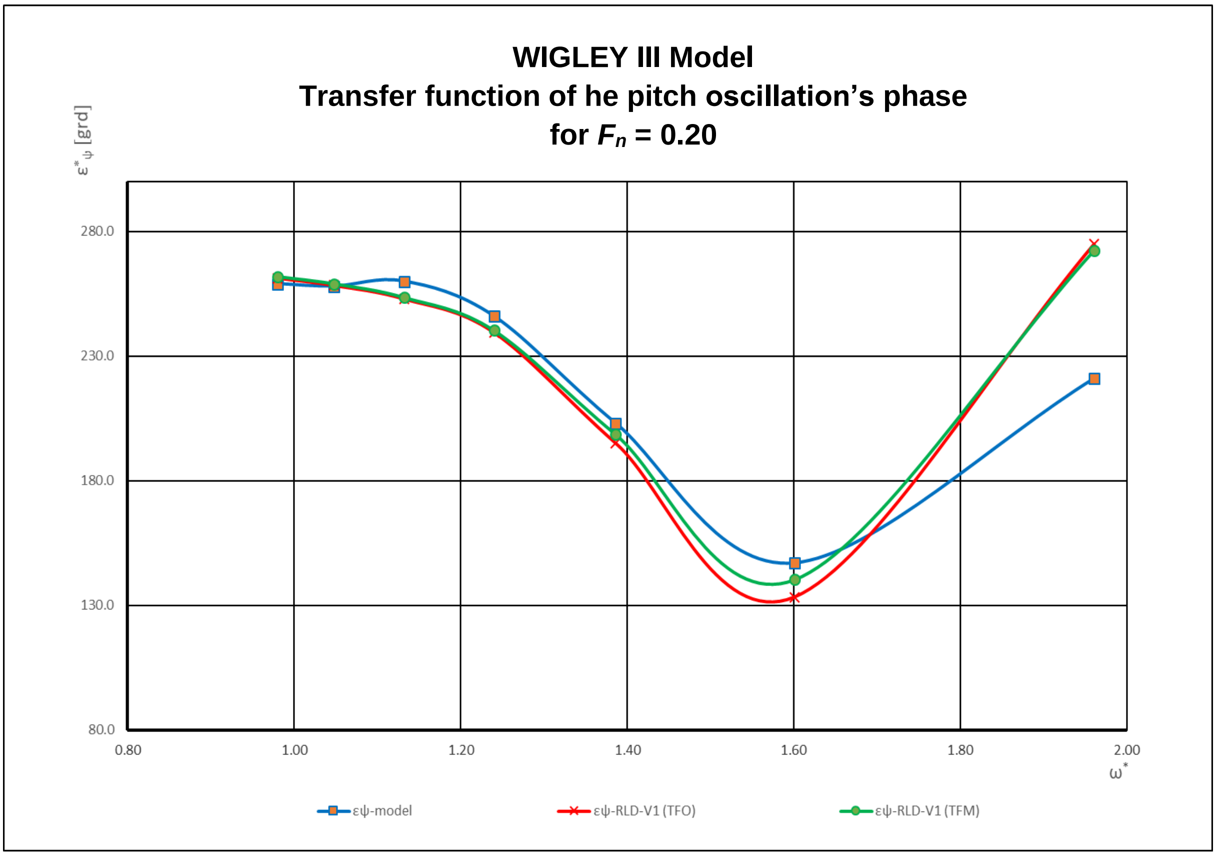


Fig. 2.3.5 – Dynamic response for vertical oscillations at F_n = 0.20.
The measurements on the Wigley III model are taken from [66]



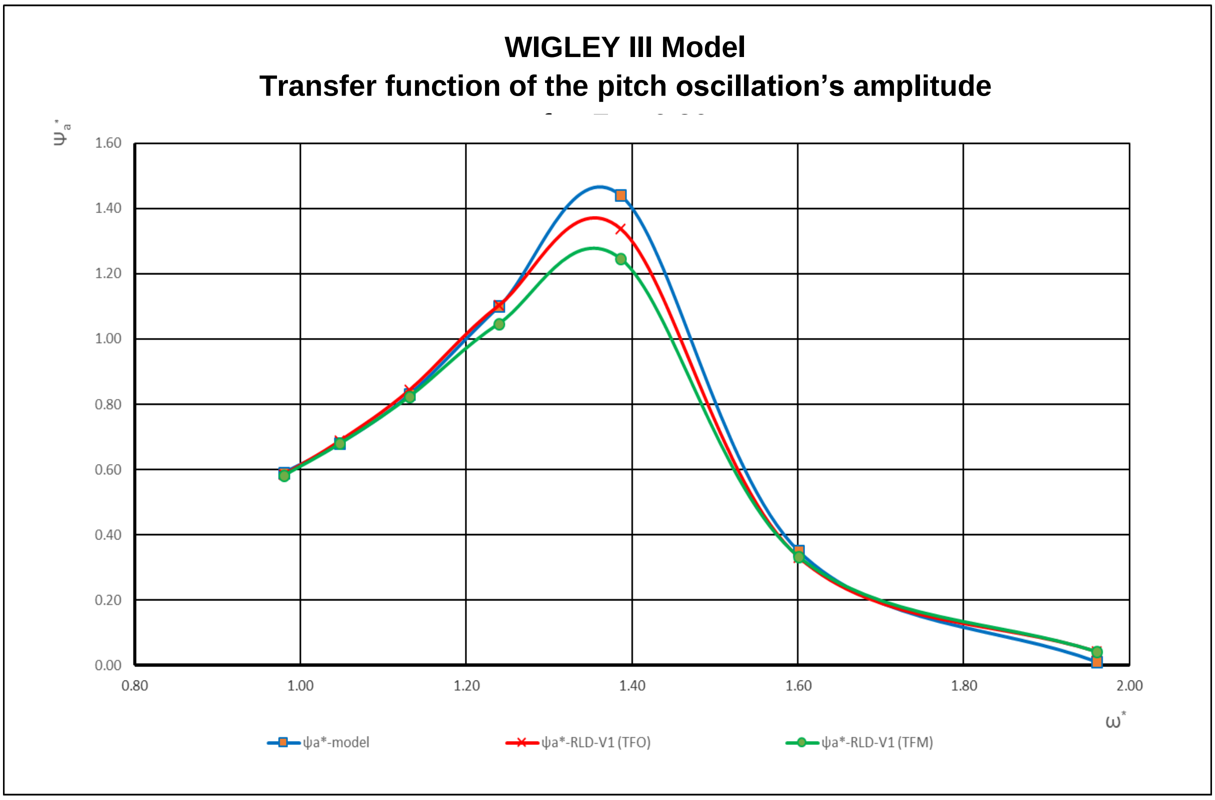


Fig. 2.3.6 – Dynamic response for pitch oscillations at F_n = 0.20.
The measurements on the Wigley III model are taken from [66]

The verification of the method and the program was also carried out by comparative calculations with the measurements presented in the works [68] and [69], made in 2009 on the Seatech-D model (representing a RoPax ship at 1:39.024 scale), within the LAINE project carried out by VTT in Merike (Finland) in collaboration with the Finnish Agency for Technology and Innovation (TEKES), Aker Shipyards Group, Technip Offshore Finland, Finnish Navy and SWECO Marine

This model, shown in Fig. 2.3.7, has the following main features:

Maximum length, Lmax ………………………………	4,4000 m
Length, L …………………..……………………….…	4,0000 m
Breadth, B ……………………………………..…..	0,6400 m
Draught, T ……………………………………..…..	0,1560 m
Trim, ψ_o …………………..…..……………………...	0,0000 grd
Distance between couples, *δ* ………….……………..	0,2027
Displacement, Δ …………………………………......	0,2300 t
Block coefficient, C_B …..………..	0,5500
Abscissa of ship's center of gravity, XG	1,9205 m
Vertical coordinate of the center of gravity, KG …….	0,1700 m
Radius of inertia for pitch, R_Y ……………………….....	1,0000 m

Fig. 2.3.7 – Seatech-D model during the tests [68]

The measurement of the bending moment was performed at couple 8, and the shear force measurement was performed at couple 13.

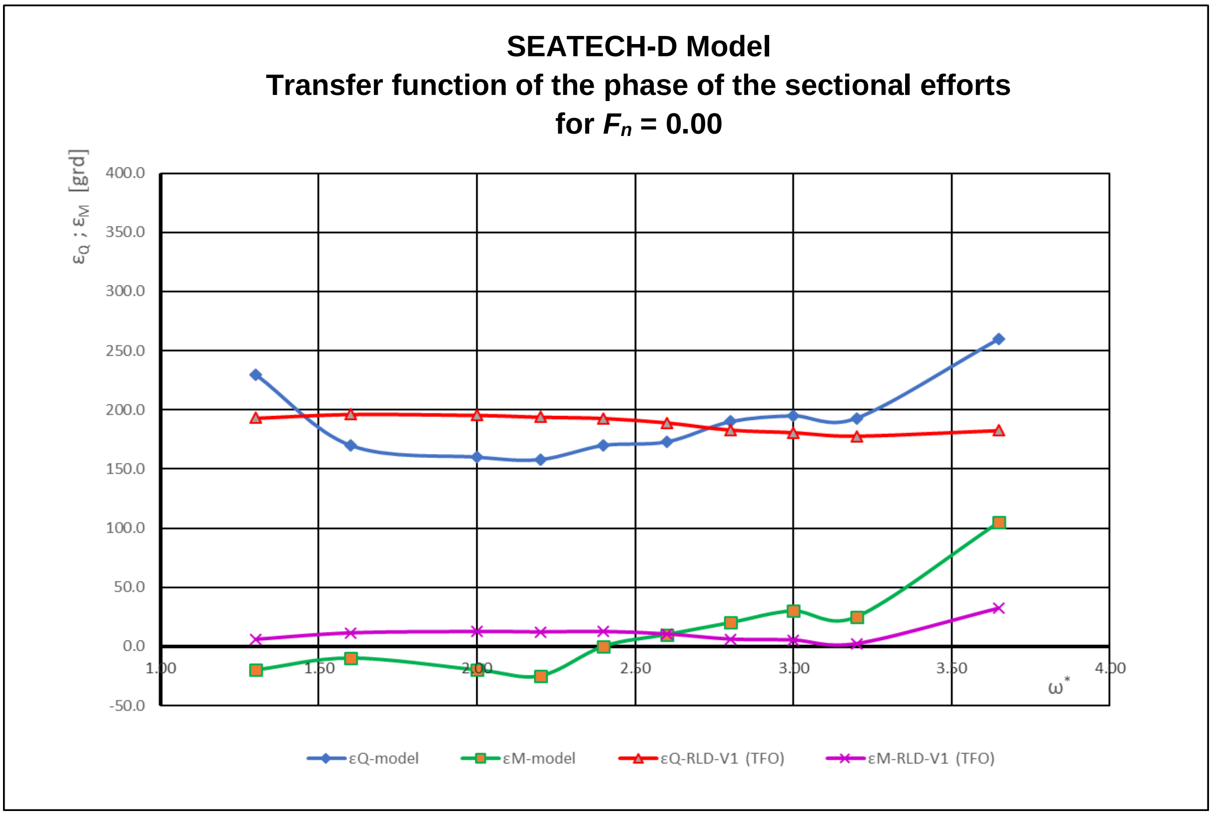
Comparison of the results of the calculations with the measurements made on the Seatech-D experimental model was performed in different navigation regimes characterized by the Froude F_n number, using the dimensionless transfer functions for the transfer functions for the amplitude of the shear force and of the bending moment at the measuring points, defined by the relations :

$$Q_a^* = \frac{2 \cdot Q_a}{\rho \cdot g \cdot h_v \cdot B \cdot L} \tag{2.3.89}$$

$$M_a^* = \frac{2 \cdot M_a}{\rho \cdot g \cdot h_v \cdot B \cdot L^2} \tag{2.3.90}$$

as a dynamic response to the action of the head waves, as well as the phases of these sectional efforts relative to the crest of the wave considered to be at the ship's center of gravity.

Figures 2.3.8 - 2.3.10 graphically show the results of the calculations according to the presented method and the measurements made on the Seatech-D model, at corresponding speeds in which F_n had the values 0 and 0.25, on regular waves of 0.048 m height and the relative pulsation ranging between 1.3 and 3.65.



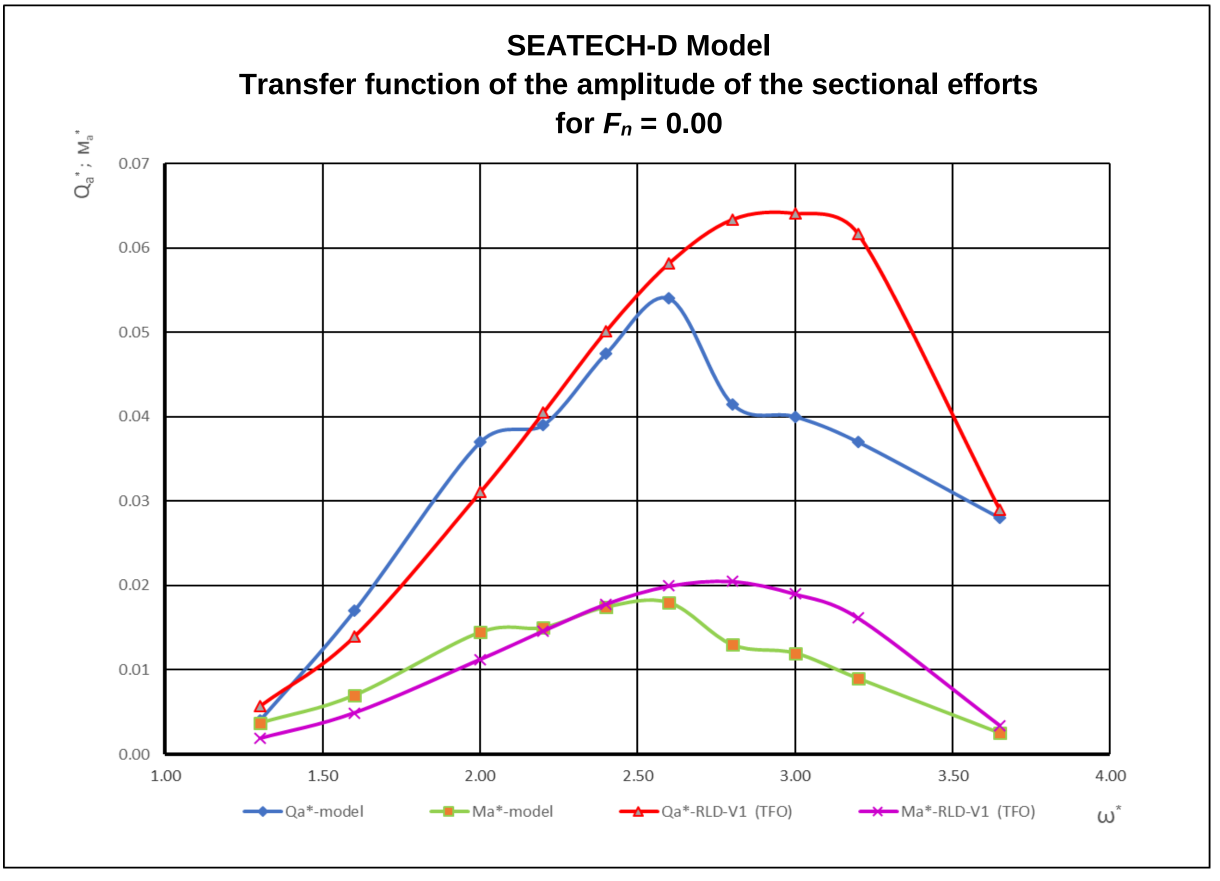
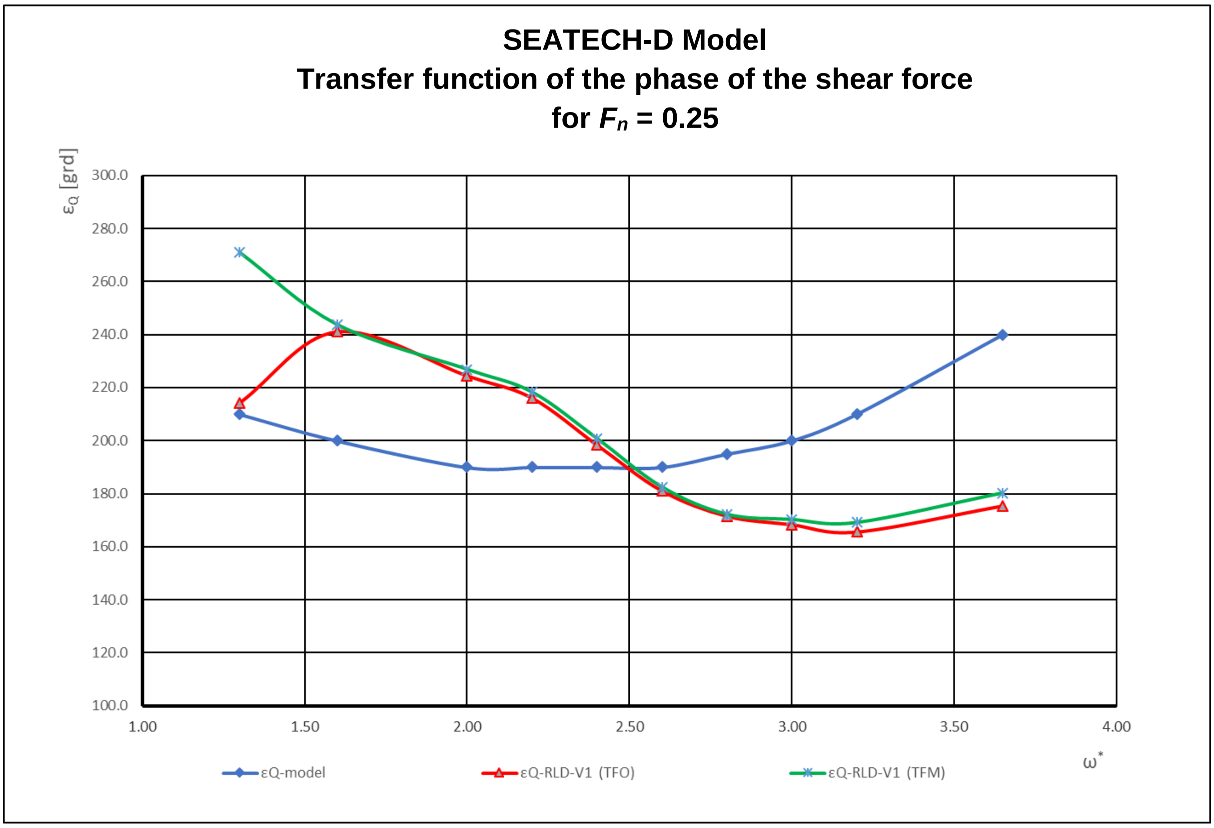


Fig. 2.3.8 – Dynamic response for sectional efforts at F_n = 0.00.
The measurements on the Seatech-D model are taken from [68]



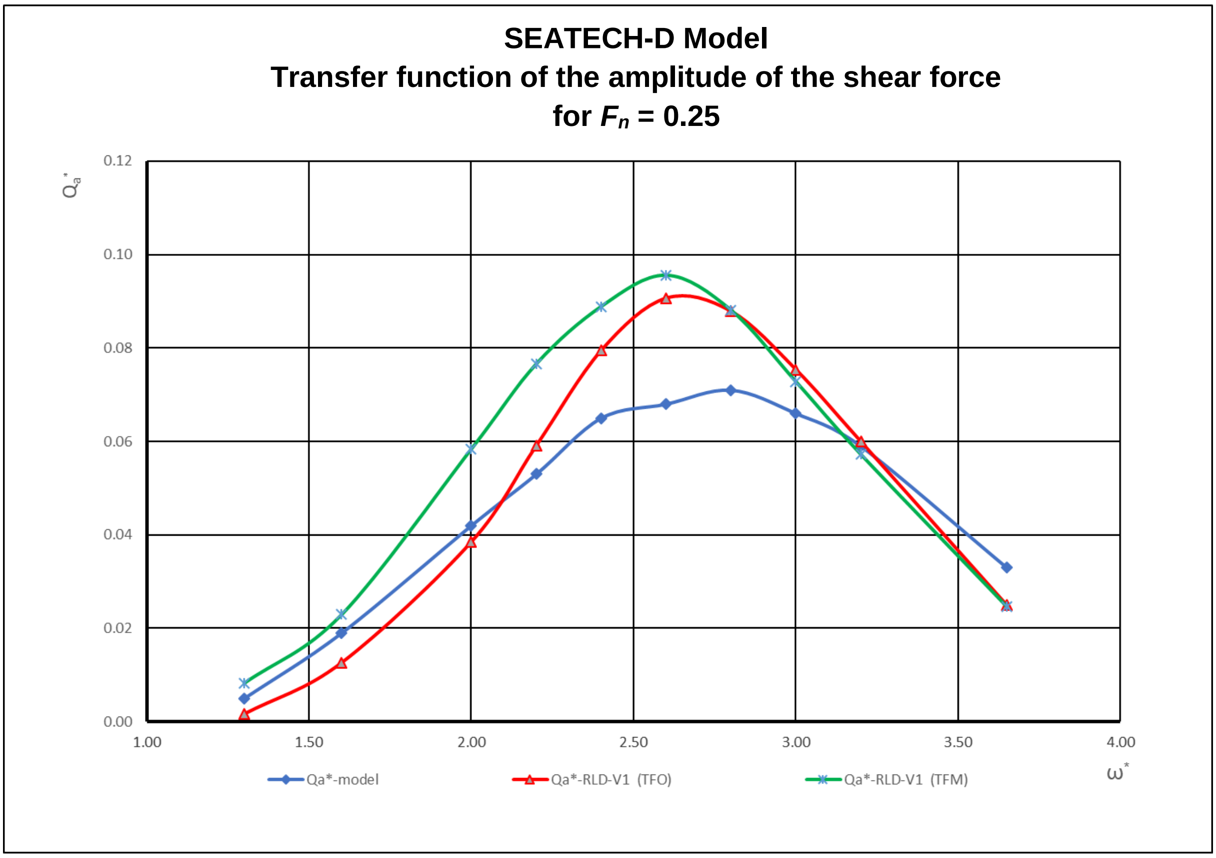
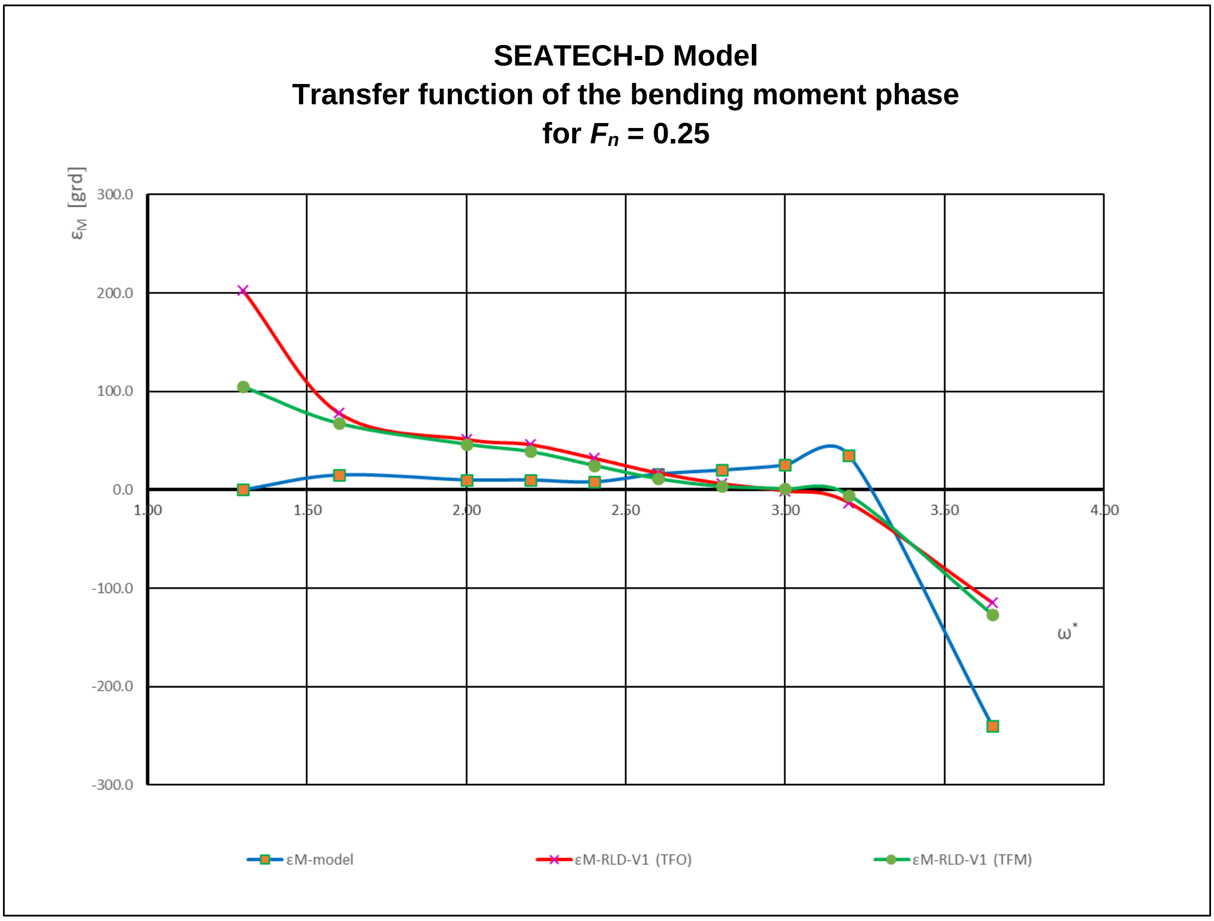


Fig. 2.3.9 – Dynamic response for shear force at F_n = 0.25.
The measurements on the Seatech-D model are taken from [68]



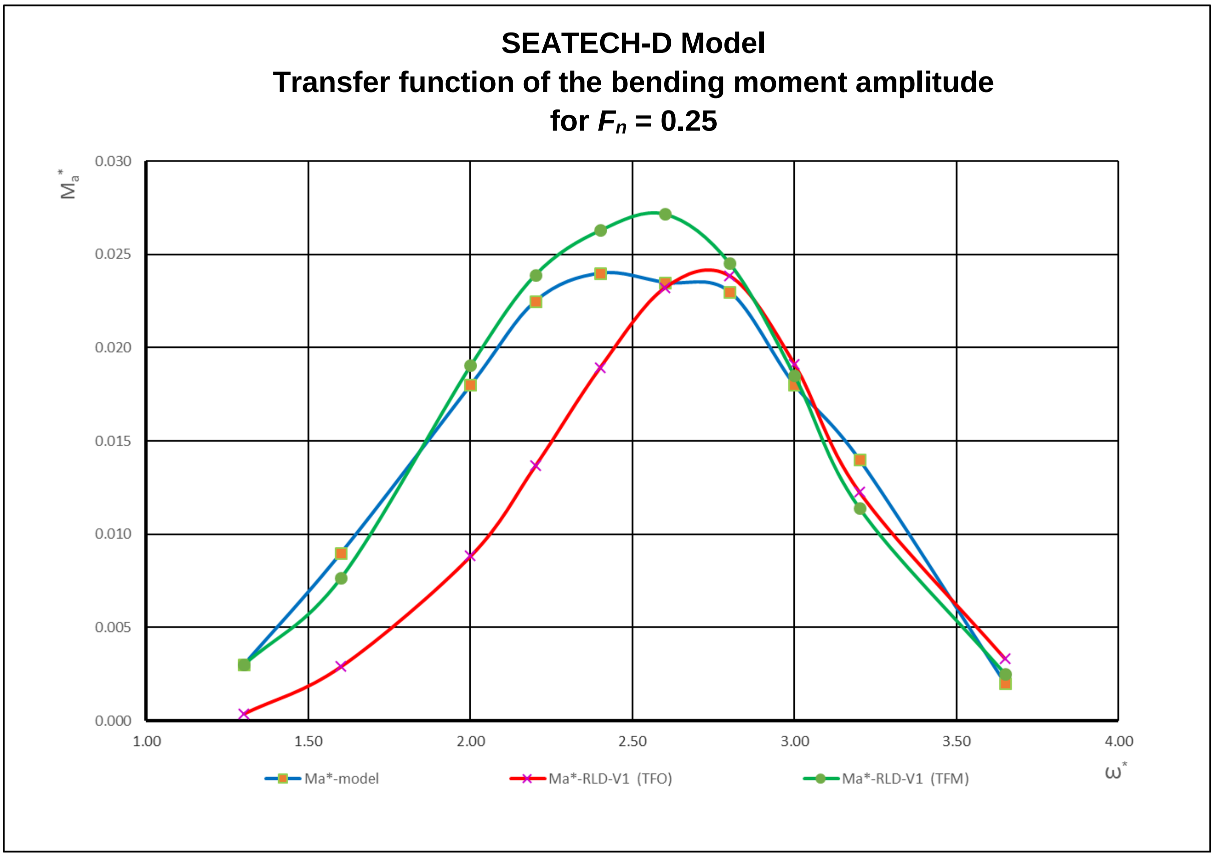


Fig. 2.3.10 – Dynamic response for bending moment at F_n = 0.25.
The measurements on the Seatech-D model are taken from [68]

In addition, verification of the calculation method and the RLD-V1 program was also performed by comparing the results of the calculations with the measurements made on a test model, presented in the paper [70]. The model was made of brass and has the configuration shown in Fig. 2.3.11, having the following characteristics:

L = 6,00 m
B = 0,80 m
D = 0,51 m
T_{max} = 0,34 m
C_B = 0,83
W_{punte} = 225 cm^3

Measurements were made in the Mejiro test basin from Tokyo, by towing the model at the draft of 0.20 m, on regular cosine waves of 6.0 m length, 0.23 m height, 1/26 gradient, and 2s, and at three speeds: 0.0, 1.39 and 2.8 m/s. The stresses measurements were made on the deck at the middle of the model. The cosine wave, equivalent of the cosine wave created in the Mejiro basin, has a height of 0.220 m.

In this case, for the verification of the method and RLD-V1 program, it was directly analyzed in time domain variation of the deck stresses at the middle of the model.

These variations are graphically shown in Fig. 2.3.12–2.3.14 for the three regimes of navigation.

2.3.5 Comments and conclusions

The RLD-V1 program for the linear computation of the ship's oscillation parameters and additional sectional efforts induced in its hull by head waves, is a personal achievement and represent a useful tool for design and research activities to improve the construction safety of the ships.

The program was developed based on the classical method presented in Works [58], [59] and [60], using " Ordinary Strip Theory" and "Modified Strip Theory" developed by the author in a specific way to automate calculations, introducing several new considerations:

- delimitation of the independent action of the hydrostatic pressures acting in still water from the those additional, induced by the waves which are affected by the Smith effect;
- considering the ship as a rigid one on which all the static and dynamic loads acts, and separating of static actions from dynamic actions;
- determining of the dynamic equilibrium equations at moments defined by the relation (2.3.79);
- determining of the additional masses of water, the amortization and the Smith effect corresponding to these moments, so that the matrix of the linear system of equilibrium equations is no longer symmetrical as it is commonly shown in the literature;
- time calculation and graphical display of the ship’s movements and sectional efforts diagram.

The calculation method presented in 2.3.2 and the RLD-V1 program has been verified on the three models presented in 2.3.4, noting that the results of the calculations are consistent with the measurements on those models, the deviations being generally below 30% and only in isolated cases, such as resonance zones, this limit is exceeded. These deviations are justified by the complexity of the ship's hydrodynamics on the waves, in which alongside the ship participate also masses of additional water which are difficult to accurately estimate, and the damping phenomena are equally difficult to determine with precision. Also, the weight distribution of the models was adopted without enough data. However, the deviations are like those presented in the literature and accepted as reasonable, so the RLD-V1 method and program can be considered to provide results that can be considered in the ship design field studies. Noting that, based on this method, maximum calculated sectional efforts have values that cover those resulting from measurements on models, which means that the calculated values are reliable in the sense that they will not be exceeded. It is recommended that, when assessing the sectional efforts, the calculations be made with both variants of the method.

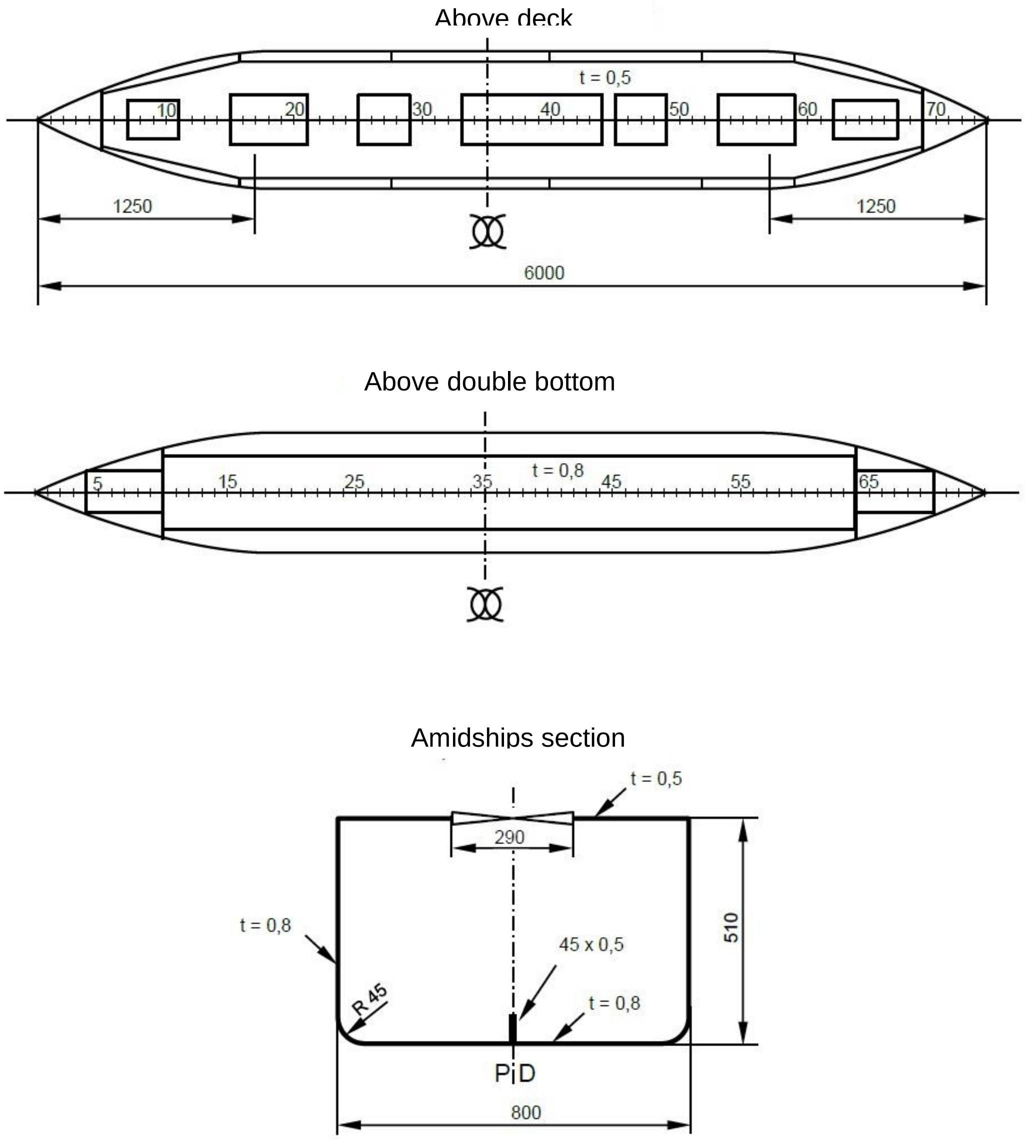


Fig. 2.3.11 – Configuration of the test model presented in paper [70]

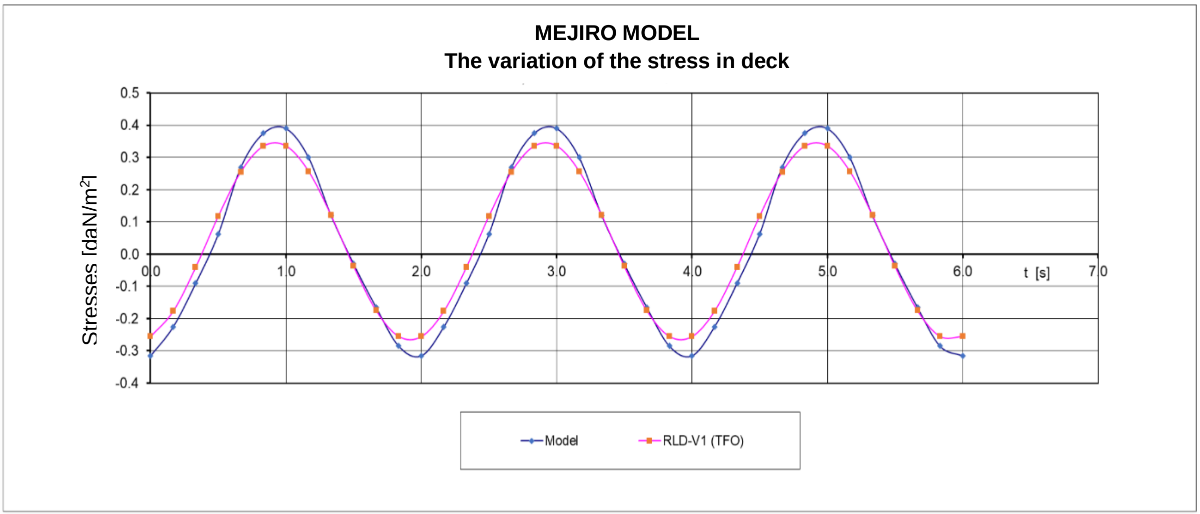


Fig. 2.3.12 –The time variation of the stress in deck, when the model's speed is 0. The measurements on the Mejiro model are taken from [70]

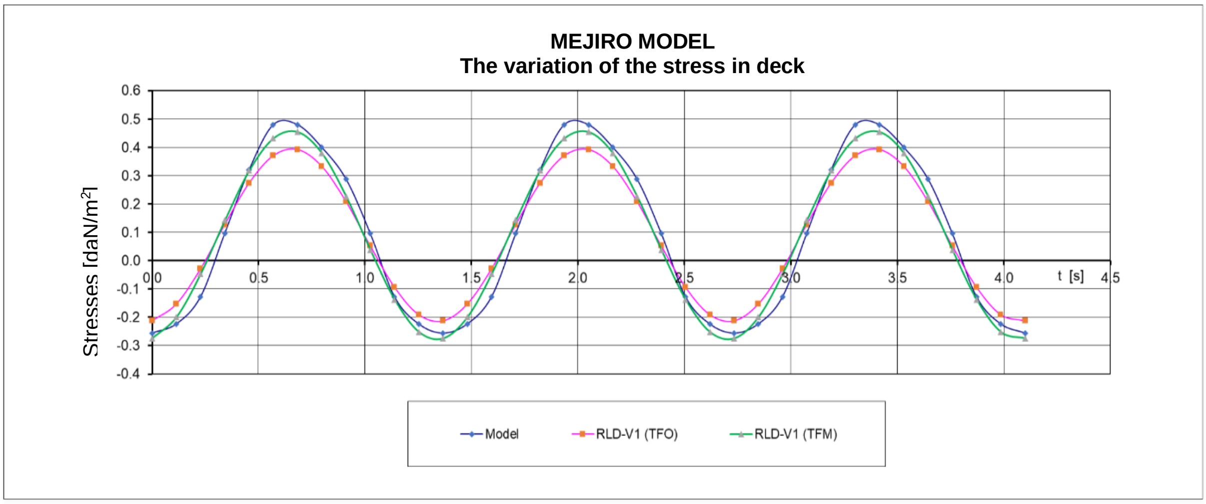


Fig. 2.3.13 – The time variation of the stress in deck, when the model's speed is 1.39 m/s. The measurements on the Mejiro model are taken from [70]

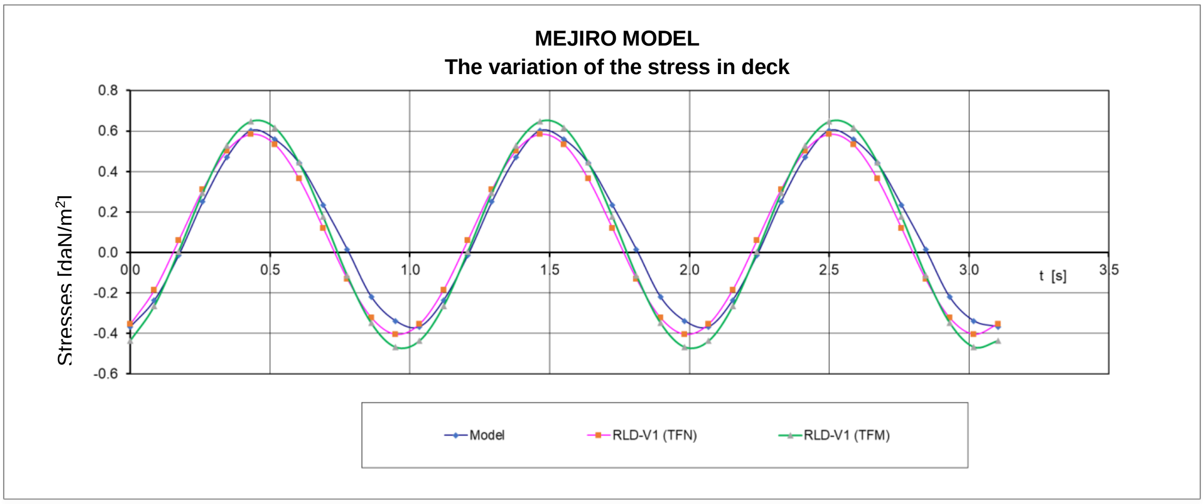


Fig. 2.3.14 – The time variation of the stress in deck, when the model's speed is 2.80 m/s. The measurements on the Mejiro model are taken from [70]

2.4 The program for non-linear calculation of the ship's oscillation parameters and of the additional sectional efforts in its hull induced by the head wave, considering linear damping

2.4.1 Program object and destination

The program allows calculation by non-linear method of the ship's oscillation parameters as well as the additional sectional efforts in its hull induced by the head wave, considering linear damping.

2.4.2 Non-linear calculation method considering linear damping

The solutions of the linear system (2.3.78) are determined at the moments indicated by the relation (2.3.79), but between these moments, the distribution of the additional masses, damping and pressures along the ship length continuously change, as a function which can be considered that cosine varies, following the wave profile, as shown in 2.3.2. At the same time, the ship's oscillations ζ and ψ also contribute to the change of these distributions along the ship length so that the ship's dynamics on the waves is actually non-linear, as proved in the diagram of Fig. 2.4.1 taken from [68] (in which is presented the time variation of the bending moment at amidships of a model, determined by measurements and by linear calculation), but can be linearized on short time intervals.

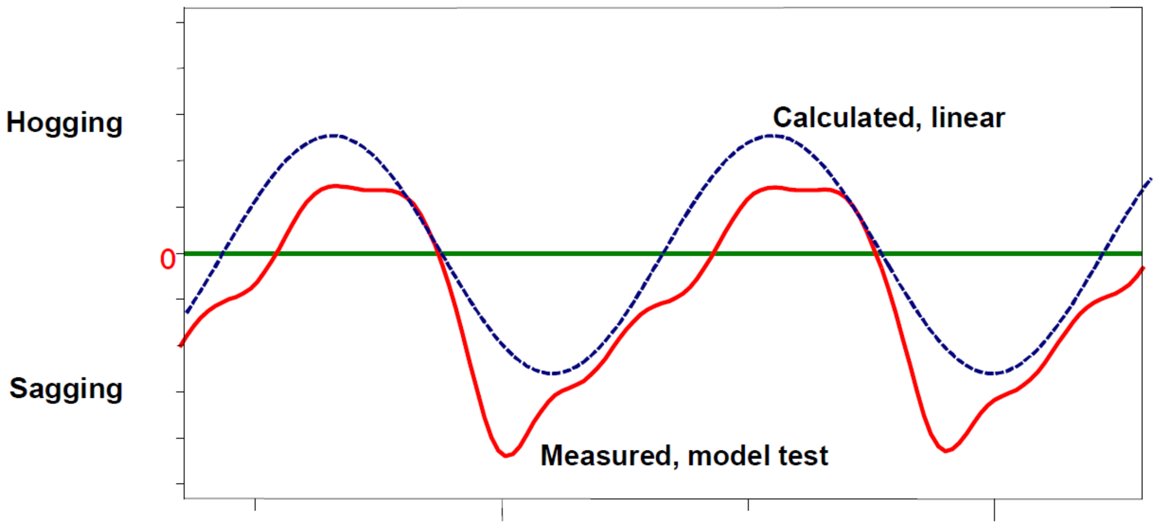


Fig. 2.4.1 – The time variation of the bending moment at model amidships, determined by measurements and linear calculation [68]

For such a short time interval, starting from relations (2.3.51) and (2.3.52), we obtain a system of two differential equations, like the system (2.3.53), having the form:

$$\begin{cases} A_{\zeta\zeta}\cdot\ddot{\zeta}+B_{\zeta\zeta}\cdot\dot{\zeta}+C_{\zeta\zeta}\cdot\zeta+A_{\zeta\psi}\cdot\ddot{\psi}+B_{\zeta\psi}\cdot\dot{\psi}+C_{\zeta\psi}\cdot\psi=F_V(\zeta,\psi,t) \\ A_{\psi\zeta}\cdot\ddot{\zeta}+B_{\psi\zeta}\cdot\dot{\zeta}+C_{\psi\zeta}\cdot\zeta+A_{\psi\psi}\cdot\ddot{\psi}+B_{\psi\psi}\cdot\dot{\psi}+C_{\psi\psi}\cdot\psi=M_V(\zeta,\psi,t) \end{cases} \tag{2.4.1}$$

where, however, the coefficients and free terms are determined by the formulas:

$$A_{\zeta\zeta}(\zeta,\psi,t)=\int_0^L [m(x)+M_{33}(x,\zeta,\psi,t)+\frac{V}{\omega_e^2}\cdot\frac{dN_{33}(x,\zeta,\psi,t)}{dx}]\cdot dx \tag{2.4.2}$$

$$B_{\zeta\zeta}(\zeta,\psi,t)=\int_0^L [N_{33}(x,\zeta,\psi,t)-V\cdot\frac{dM_{33}(x,\zeta,\psi,t)}{dx}]\cdot dx \tag{2.4.3}$$

$$C_{\zeta\zeta}(\zeta,\psi,t)=2\cdot\rho\cdot g\cdot\int_0^L y(x,\zeta,\psi,t)\cdot dx \tag{2.4.4}$$

$$A_{\zeta\psi}(\zeta,\psi,t)=-\int_0^L x\cdot[m(x)+M_{33}(x,\zeta,\psi,t)+\frac{V}{\omega_e^2}\cdot\frac{dN_{33}(x,\zeta,\psi,t)}{dx}]\cdot dx \tag{2.4.5}$$

$$B_{\zeta\psi}(\zeta,\psi,t)=-\int_0^L \{-2\cdot V\cdot[M_{33}(x,\zeta,\psi,t)+\frac{V}{\omega_e^2}\cdot\frac{dN_{33}(x,\zeta,\psi,t)}{dx}]+ \\ +x\cdot[N_{33}(x,\zeta,\psi,t)-V\cdot\frac{dM_{33}(x,t)}{dx}]\}dx \tag{2.4.6}$$

$$C_{\zeta\psi}(\zeta,\psi,t)=-\int_0^L \{2\cdot\rho\cdot g\cdot x\cdot y(x,\zeta,\psi,t)- \\ -V\cdot[N_{33}(x,\zeta,\psi,t)-V\cdot\frac{dM_{33}(x,\zeta,\psi,t)}{dx}]\}\cdot dx \tag{2.4.7}$$

$$A_{\psi\zeta}(\zeta,\psi,t)=\int_0^L x\cdot[m(x)+M_{33}(x,\zeta,\psi,t)+\frac{V}{\omega_e^2}\cdot\frac{dN_{33}(x,\zeta,\psi,t)}{dx}]\cdot dx \tag{2.4.8}$$

$$B_{\psi\zeta}(\zeta,\psi,t)=\int_0^L x\cdot[N_{33}(x,\zeta,\psi,t)-V\cdot\frac{dM_{33}(x,\zeta,\psi,t)}{dx}]\cdot dx \tag{2.4.9}$$

$$C_{\psi\zeta}(\zeta,\psi,t)=2\cdot\rho\cdot g\cdot\int_0^L x\cdot y(x,\zeta,\psi,t)\cdot dx \tag{2.4.10}$$

$$A_{\psi\psi}(\zeta,\psi,t)=-\int_0^L x^2\cdot[m(x)+M_{33}(x,\zeta,\psi,t)+\frac{V}{\omega_e^2}\cdot\frac{dN_{33}(x,\zeta,\psi,t)}{dx}]\cdot dx \tag{2.4.11}$$

$$B_{\psi\psi}(\zeta,\psi,t)=-\int_0^L x\cdot\{-2\cdot V\cdot[M_{33}(x,\zeta,\psi,t)+\frac{V}{\omega_e^2}\cdot\frac{dN_{33}(x,\zeta,\psi,t)}{dx}]+ \\ +x\cdot[N_{33}(x,\zeta,\psi,t)-V\cdot\frac{dM_{33}(x,\zeta,\psi,t)}{dx}]\}\cdot dx \tag{2.4.12}$$

$$C_{\psi\psi}(\zeta,\psi,t)=-\int_0^L \{2\cdot\rho\cdot g\cdot x^2\cdot y(x,\zeta,\psi,t)- \\ -V\cdot x\cdot[N_{33}(x,\zeta,\psi,t)-V\cdot\frac{dM_{33}(x,\zeta,\psi,t)}{dx}]\}\cdot dx \tag{2.4.13}$$

$$F_V(\zeta,\psi,t)=-\omega^2\cdot\frac{h_V}{2}\cdot\int_0^L f_1(x,\zeta,\psi,t)\cdot[M_{33}(x,\zeta,\psi,t)+$$

$$+\frac{V}{\omega_e^2}\cdot\frac{dN_{33}(x,\zeta,\psi,t)}{dx}]\cdot\cos(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)\cdot dx-$$

$$-\omega\cdot\frac{h_V}{2}\cdot\int_0^L f_1(x,\zeta,\psi,t)\cdot[N_{33}(x,\zeta,\psi,t)-$$

$$-V\cdot\frac{dM_{33}(x,\zeta,\psi,t)}{dx}]\cdot\sin(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)\cdot dx+$$

$$+\rho\cdot g\cdot h_V\cdot\int_0^L f_1(x,\zeta,\psi,t)\cdot y(x,\zeta,\psi,t)\cdot\cos(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)\cdot dx \quad (2.4.14)$$

$$M_V(\zeta,\psi,t)=-\omega^2\cdot\frac{h_V}{2}\cdot\int_0^L x\cdot f_1(x,\zeta,\psi,t)\cdot[M_{33}(x,\zeta,\psi,t)+$$

$$+\frac{V}{\omega_e^2}\cdot\frac{dN_{33}(x,\zeta,\psi,t)}{dx}]\cdot\cos(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)\cdot dx-$$

$$-\omega\cdot\frac{h_V}{2}\cdot\int_0^L x\cdot f_1(x,\zeta,\psi,t)\cdot[N_{33}(x,\zeta,\psi,t)-$$

$$-V\cdot\frac{dM_{33}(x,\zeta,\psi,t)}{dx}]\cdot\sin(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)\cdot dx+$$

$$+\rho\cdot g\cdot h_V\cdot\int_0^L x\cdot f_1(x,\zeta,\psi,t)\cdot y(x,\zeta,\psi,t)\cdot\cos(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)\cdot dx \quad (2.4.15)$$

$$f_1(x,\zeta,\psi,t)=1-\frac{2\cdot k}{b_m(x,\zeta,\psi,t)}\cdot$$

$$\cdot\int_{-T(x,\zeta,\psi,t)}^{\varsigma_V(x,t)} y(x,z,\zeta,\psi,t)\cdot e^{k[z-\varsigma_V(x,t)]}\cdot dz \quad (2.4.16)$$

$$b_m(x,\zeta,\psi,t)=\frac{\int_{0(x,\zeta,\psi,t)}^{h_V/2} y(x,z,\zeta,\psi,t)\cdot dz}{h_V} \quad (2.4.17)$$

$$T(x,\zeta,\psi,t)=T_o-\zeta(t)+x\cdot[\psi_o-\psi(t)]+\zeta_V(x,t) \quad (2.4.18)$$

Quota $0(x,\zeta,,\psi,t)$ is the surface elevation of the still water to which the ship relates and which changes over time, depending on the ship's oscillations.

Although the system of the differential equations of order two (2.4.1) is nonlinear, it can be assumed that, on short time intervals, the additional masses, the damping and the Smith effect are linear, so that this system is also linear at such intervals, and its stabilized solution varies according to a cosinusoidal rule as those of the external loads, so it can be admitted that it is given by the relations (2.3.72) and (2.3.73).

By replacing these relations in (2.4.1) and grouping by cos and sin, we obtain the system of two equations in which the unknowns are $\zeta_1,\ \zeta_2,\ \psi_1,\ \psi_2$:

$$\begin{cases} [(C_{\zeta\zeta}-\omega_e^2 A_{\zeta\zeta})\cdot\zeta_1+\omega_e\cdot B_{\zeta\zeta}\cdot\zeta_2+(C_{\zeta\psi}-\omega_e^2 A_{\zeta\psi})\cdot\psi_1+\omega_e\cdot B_{\zeta\psi}\cdot\psi_2]\cdot\cos(\omega_e\cdot t)+ \\ +[-\omega_e\cdot B_{\zeta\zeta}\cdot\zeta_1+(C_{\zeta\zeta}-\omega_e^2 A_{\zeta\zeta})\cdot\zeta_2-\omega_e\cdot B_{\zeta\psi}\cdot\psi_1+(C_{\zeta\psi}-\omega_e^2 A_{\zeta\psi})\cdot\psi_2]\cdot\sin(\omega_e\cdot t)= \\ =F_V(\zeta_1,\zeta_2,\psi_1,\psi_2,t) \\ [(C_{\psi\zeta}-\omega_e^2 A_{\psi\zeta})\cdot\zeta_1+\omega_e\cdot B_{\psi\zeta}\cdot\zeta_2+(C_{\psi\psi}-\omega_e^2 A_{\psi\psi})\cdot\psi_1+\omega_e\cdot B_{\psi\psi}\cdot\psi_2]\cdot\cos(\omega_e\cdot t)+ \\ +[-\omega_e\cdot B_{\psi\zeta}\cdot\zeta_1+(C_{\psi\zeta 1}-\omega_e^2 A_{\psi\zeta 1})\cdot\zeta_2+(C_{\psi\psi 1}-\omega_e^2 A_{\xi\psi 1})\cdot\psi_2-\omega_e\cdot B_{\psi\psi}\cdot\psi_1]\cdot\sin(\omega_e\cdot t)= \\ =M_V(\zeta_1,\zeta_2,\psi_1,\psi_2,t) \end{cases} \tag{2.4.19}$$

where:

$$\begin{aligned} F_V(\zeta_1,\zeta_2,\psi_1,\psi_2,t) &= -\omega^2\cdot\frac{h_V}{2}\cdot\int_0^L f_1(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)\cdot[M_{33}(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)+ \\ &+\frac{V}{\omega_e^2}\cdot\frac{dN_{33}(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)}{dx}]\cdot\cos(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)\cdot dx- \\ &-\omega\cdot\frac{h_V}{2}\cdot\int_0^L f_1(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)\cdot[N_{33}(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)- \\ &-V\cdot\frac{dM_{33}(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)}{dx}]\cdot\sin(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)\cdot dx+ \\ &+\rho\cdot g\cdot h_V\cdot\int_0^L f_1(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)\cdot y(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)\cdot\cos(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)\cdot dx \end{aligned} \tag{2.4.20}$$

$$\begin{aligned} M_V(\zeta_1,\zeta_2,\psi_1,\psi_2,t) &= -\omega^2\cdot\frac{h_V}{2}\cdot\int_0^L x\cdot f_1(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)\cdot[M_{33}(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)+ \\ &+\frac{V}{\omega_e^2}\cdot\frac{dN_{33}(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)}{dx}]\cdot\cos(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)\cdot dx- \\ &-\omega\cdot\frac{h_V}{2}\cdot\int_0^L x\cdot f_1(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)\cdot[N_{33}(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)- \\ &-V\cdot\frac{dM_{33}(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)}{dx}]\cdot\sin(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)\cdot dx+ \\ &+\rho\cdot g\cdot h_V\cdot\int_0^L x\cdot f_1(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)\cdot y(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)\cdot\cos(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)\cdot dx \end{aligned} \tag{2.4.21}$$

$$f_1(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)=1-\frac{2\cdot k}{b_m(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)}\cdot$$

$$\cdot\int_{-T(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)}^{\varsigma_V(x,t)} y(x,z,\zeta_1,\zeta_2,\psi_1,\psi_2,t)\cdot e^{k[z-\varsigma_V(x,t)]}\cdot dz \tag{2.4.22}$$

$$b_m(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)=\frac{\int_{0(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)}^{h_V/2} y(x,z,\zeta_1,\zeta_2,\psi_1,\psi_2,t)\cdot dz}{h_V} \tag{2.4.23}$$

$$T(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)=T_o-[\zeta_1\cdot\cos(\omega_e\cdot t)+\zeta_2\cdot\sin(\omega_e\cdot t)]+$$
$$+x\cdot\{\psi_o-[\psi_1\cdot\cos(\omega_e\cdot t)+\psi_2\cdot\sin(\omega_e\cdot t]\}+$$
$$+\frac{h_V}{2}\cdot\cos(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega\cdot t) \tag{2.4.24}$$

Because there are only two equations, the 4 unknowns ζ_1, ζ_2, ψ_1, ψ_2, are determined by balancing the ship at the ends of the time intervals, defined by the formulas:

$$t_p=\frac{2\cdot\pi}{\omega_e\cdot N_T}\cdot p$$
$$t_{p+1}=\frac{2\cdot\pi}{\omega_e\cdot N_T}\cdot(p+1) \tag{2.4.25}$$

where :

N_T – no. of the time intervals in which the oscillation period is divided,

for each interval *p*, resulting a system of 4 non-linear equations, with the unknowns ζ_{p1}, ζ_{p2}, ψ_{p1}, ψ_{p2}, which can be written under the following matrix form:

$$A_p(X_p)\cdot X_p=F_p(X_p) \tag{2.4.26}$$

where:

$$A_p(X_p)=\begin{bmatrix} A_{p11} & A_{p12} & A_{p13} & A_{p14} \\ A_{p21} & A_{p22} & A_{p23} & A_{p24} \\ A_{p31} & A_{p32} & A_{p33} & A_{p34} \\ A_{p41} & A_{p42} & A_{p43} & A_{p44} \end{bmatrix} \tag{2.4.27}$$

$$X_p=\{\zeta_{p1}\quad \zeta_{p2}\quad \psi_{p1}\quad \psi_{p2}\}^T \tag{2.4.28}$$

$$F_p(X_p)=\{F_{Vp1}(X_p,t_p)\quad F_{Vp2}(X_p,t_{p+1})\quad M_{Vp1}(X_p,t_p)\quad M_{Vp2}(X_p,t_{p+1})\}^T \tag{2.4.29}$$

$$A_{p11}(X_p)=(C_{\zeta\zeta p}-\omega_e^2A_{\zeta\zeta p})\cdot\cos(\omega_e\cdot t_p)-\omega_e\cdot B_{\zeta\zeta p}\cdot\sin(\omega_e\cdot t_p) \tag{2.4.30}$$

$$A_{p12}(X_p)=(C_{\zeta\zeta p}-\omega_e^2A_{\zeta\zeta p})\cdot\sin(\omega_e\cdot t_p)+\omega_e\cdot B_{\zeta\zeta p}\cdot\cos(\omega_e\cdot t_p) \tag{2.4.31}$$

$$A_{p13}(X_p)=(C_{\zeta\psi p}-\omega_e^2A_{\zeta\psi p})\cdot\cos(\omega_e\cdot t_p)-\omega_e\cdot B_{\zeta\psi p}\cdot\sin(\omega_e\cdot t_p) \tag{2.4.32}$$

$$A_{p14}(X_p) = (C_{\zeta\psi p} - \omega_e^2 A_{\zeta\psi p}) \cdot \sin(\omega_e \cdot t_p) + \omega_e \cdot B_{\zeta\psi p} \cdot \cos(\omega_e \cdot t_p) \quad (2.4.33)$$

$$A_{p21}(X_p) = (C_{\zeta\zeta p+1} - \omega_e^2 A_{\zeta\zeta p+1}) \cdot \cos(\omega_e \cdot t_{p+1}) - \omega_e \cdot B_{\zeta\zeta p+1} \cdot \sin(\omega_e \cdot t_{p+1}) \quad (2.4.34)$$

$$A_{p22}(X_p) = (C_{\zeta\zeta p+1} - \omega_e^2 A_{\zeta\zeta p+1}) \cdot \sin(\omega_e \cdot t_{p+1}) + \omega_e \cdot B_{\zeta\zeta p+1} \cdot \cos(\omega_e \cdot t_{p+1}) \quad (2.4.35)$$

$$A_{p23}(X_p) = (C_{\zeta\psi p+1} - \omega_e^2 A_{\zeta\psi p+1}) \cdot \cos(\omega_e \cdot t_{p+1}) - \omega_e \cdot B_{\zeta\psi p+1} \cdot \sin(\omega_e \cdot t_{p+1}) \quad (2.4.36)$$

$$A_{p24}(X_p) = (C_{\zeta\psi p+1} - \omega_e^2 A_{\zeta\psi p+1}) \cdot \sin(\omega_e \cdot t_{p+1}) + \omega_e \cdot B_{\zeta\psi p+1} \cdot \cos(\omega_e \cdot t_{p+1}) \quad (2.4.37)$$

$$A_{p31}(X_p) = (C_{\psi\zeta p} - \omega_e^2 A_{\psi\zeta p}) \cdot \cos(\omega_e \cdot t_p) - \omega_e \cdot B_{\psi\zeta p} \cdot \sin(\omega_e \cdot t_p) \quad (2.4.38)$$

$$A_{p32}(X_p) = (C_{\psi\zeta p} - \omega_e^2 A_{\psi\zeta p}) \cdot \sin(\omega_e \cdot t_p) + \omega_e \cdot B_{\psi\zeta p} \cdot \cos(\omega_e \cdot t_p) \quad (2.4.39)$$

$$A_{p33}(X_p) = (C_{\psi\psi p} - \omega_e^2 A_{\psi\psi p}) \cdot \cos(\omega_e \cdot t_p) - \omega_e \cdot B_{\psi\psi p} \cdot \sin(\omega_e \cdot t_p) \quad (2.4.40)$$

$$A_{p34}(X_p) = (C_{\psi\psi p} - \omega_e^2 A_{\psi\psi p}) \cdot \sin(\omega_e \cdot t_p) + \omega_e \cdot B_{\psi\psi p} \cdot \cos(\omega_e \cdot t_p) \quad (2.4.41)$$

$$A_{p41}(X_p) = (C_{\psi\zeta p+1} - \omega_e^2 A_{\psi\zeta p+1}) \cdot \cos(\omega_e \cdot t_{p+1}) - \omega_e \cdot B_{\psi\zeta p+1} \cdot \sin(\omega_e \cdot t_{p+1}) \quad (2.4.42)$$

$$A_{p42}(X_p) = (C_{\psi\zeta p+1} - \omega_e^2 A_{\psi\zeta p+1}) \cdot \sin(\omega_e \cdot t_{p+1}) + \omega_e \cdot B_{\psi\zeta p+1} \cdot \cos(\omega_e \cdot t_{p+1}) \quad (2.4.43)$$

$$A_{p43}(X_p) = (C_{\psi\psi p+1} - \omega_e^2 A_{\psi\psi p+1}) \cdot \cos(\omega_e \cdot t_{p+1}) - \omega_e \cdot B_{\psi\psi p+1} \cdot \sin(\omega_e \cdot t_{p+1}) \quad (2.4.44)$$

$$A_{p44}(X_p) = (C_{\psi\psi p+1} - \omega_e^2 A_{\psi\psi p+1}) \cdot \sin(\omega_e \cdot t_{p+1}) + \omega_e \cdot B_{\psi\psi p+1} \cdot \cos(\omega_e \cdot t_{p+1}) \quad (2.4.45)$$

$$F_{Vp1}(X_p) = F_V(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_p) \quad (2.4.46)$$

$$F_{Vp2}(X_p) = F_V(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_{p+1}) \quad (2.4.47)$$

$$M_{Vp1}(X_p) = M_V(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_p) \quad (2.4.48)$$

$$M_{Vp2}(X_p) = M_V(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_{p+1}) \quad (2.4.49)$$

$$A_{\zeta\zeta p} = A_{\zeta\zeta}(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_p) \quad (2.4.50)$$

$$A_{\zeta\zeta p+1} = A_{\zeta\zeta}(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_{p+1}) \quad (2.4.51)$$

$$B_{\zeta\zeta p} = B_{\zeta\zeta}(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_p) \quad (2.4.52)$$

$$B_{\zeta\zeta p+1} = B_{\zeta\zeta}(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_{p+1}) \quad (2.4.53)$$

$$C_{\zeta\zeta p} = C_{\zeta\zeta}(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_p) \quad (2.4.54)$$

$$C_{\zeta\zeta p+1} = C_{\zeta\zeta}(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_{p+1}) \quad (2.4.55)$$

$$A_{\zeta\psi p} = A_{\zeta\psi}(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_p) \quad (2.4.56)$$

$$A_{\zeta\psi p+1} = A_{\zeta\psi}(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_{p+1}) \tag{2.4.57}$$

$$B_{\zeta\psi p} = B_{\zeta\psi}(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_p) \tag{2.4.58}$$

$$B_{\zeta\psi p+1} = B_{\zeta\psi}(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_{p+1}) \tag{2.4.59}$$

$$C_{\zeta\psi p} = C_{\zeta\psi}(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_p) \tag{2.4.60}$$

$$C_{\zeta\psi p+1} = C_{\zeta\psi}(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_{p+1}) \tag{2.4.61}$$

$$A_{\psi\zeta p} = A_{\psi\zeta}(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_p) \tag{2.4.62}$$

$$A_{\psi\zeta p+1} = A_{\psi\zeta}(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_{p+1}) \tag{2.4.63}$$

$$B_{\psi\zeta p} = B_{\psi\zeta}(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_p) \tag{2.4.64}$$

$$B_{\psi\zeta p+1} = B_{\psi\zeta}(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_{p+1}) \tag{2.4.65}$$

$$C_{\psi\zeta p} = C_{\psi\zeta}(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_p) \tag{2.4.66}$$

$$C_{\psi\zeta p+1} = C_{\psi\zeta}(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_{p+1}) \tag{2.4.67}$$

$$A_{\psi\psi p} = A_{\psi\psi}(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_p) \tag{2.4.68}$$

$$A_{\psi\psi p+1} = A_{\psi\psi}(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_{p+1}) \tag{2.4.69}$$

$$B_{\psi\psi p} = B_{\psi\psi}(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_p) \tag{2.4.70}$$

$$B_{\psi\psi p+1} = B_{\psi\psi}(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_{p+1}) \tag{2.4.71}$$

$$C_{\psi\psi p} = C_{\psi\psi}(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_p) \tag{2.4.72}$$

$$C_{\psi\psi p+1} = C_{\psi\psi}(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_{p+1}) \tag{2.4.73}$$

The non-linear system solution (2.4.26) is performed by the successive approximation method, in a version adapted to the shape of this system. The algorithm of this method consists in finding of the final solution by successive iterations, starting from a initial solution $X_p^{(0)} = \{0 \quad 0 \quad 0 \quad 0\}^T$, which is used in the calculation of the matrix $A_p(X_p^{(0)})$ and the vector $F_p(X_p^{(0)})$.

Knowing the matrix of the system and its right member, we determine the solution to the first iteration with the relation:

$$X_p^{(1)} = A_p^{(0)-1}(X_p^{(0)}) \cdot F_p(X_p^{(0)}) \tag{2.4.74}$$

Generalizing, the solution at the step *k* is determined with the relation:

$$X_p^{(k)} = A_p^{(k-1)^{-1}}(X_p^{(k-1)}) \cdot F(X_p^{(k-1)}) \tag{2.4.75}$$

the process continuing until the conditions are met:

$$\left|\zeta_p^{(k)} - \zeta_p^{(k-1)}\right| \le \varepsilon_\zeta \tag{2.4.76}$$

$$\left|\psi_p^{(k)} - \psi_p^{(k-1)}\right| \le \varepsilon_\psi \tag{2.4.77}$$

where :

$$\zeta_p^{(k)} = \zeta_{p1}^{(k)} \cdot \cos(\omega_e \cdot t_p) + \zeta_{p2}^{(k)} \cdot \sin(\omega_e \cdot t_p) \tag{2.4.78}$$

$$\zeta_p^{(k-1)} = \zeta_{p1}^{(k-1)} \cdot \cos(\omega_e \cdot t_p) + \zeta_{p2}^{(k-1)} \cdot \sin(\omega_e \cdot t_p) \tag{2.4.79}$$

$$\psi_p^{(k)} = \psi_{p1}^{(k)} \cdot \cos(\omega_e \cdot t_p) + \psi_{p2}^{(k)} \cdot \sin(\omega_e \cdot t_p) \tag{2.4.80}$$

$$\psi_p^{(k-1)} = \psi_{p1}^{(k-1)} \cdot \cos(\omega_e \cdot t_p) + \psi_{p2}^{(k-1)} \cdot \sin(\omega_e \cdot t_p) \tag{2.4.81}$$

$$\varepsilon_\zeta = h_V / 100 \tag{2.4.82}$$

$$\varepsilon_\psi = h_V / L / 100 \tag{2.4.83}$$

After finding the solution, starting from the relations (2.3.72) and (2.3.78), is calculate the additional shear force and the additional bending moment when the ship moves on waves, on each time interval *p*, along the ship's length.

The successive approximation method has the disadvantage that in the resonance zones it becomes divergent, and in this case shall be used the β-Newmark time integration method presented in [60] and [73], adapted and developed to solve the (2.4.1) system.

According to this method, the oscillation period is divided in N_T sufficiently short intervals of Δt duration, delimited by the times defined by (2.4.25), on which the acceleration is considered to be constant and equal to the arithmetic mean of the values from the ends of these intervals, so there are relations:

$$\ddot{\zeta}(t) = \frac{\ddot{\zeta}_p + \ddot{\zeta}_{p+1}}{2} \tag{2.4.84}$$

$$\dot{\zeta}_{p+1} = \dot{\zeta}_p + (\ddot{\zeta}_p + \ddot{\zeta}_{p+1}) \cdot \frac{\Delta t}{2} \tag{2.4.85}$$

$$\zeta_{p+1} = \zeta_p + \dot{\zeta}_p \cdot \Delta t + (\ddot{\zeta}_p + \ddot{\zeta}_{p+1}) \cdot \left(\frac{\Delta t}{2}\right)^2 \tag{2.4.86}$$

$$\ddot{\psi}(t) = \frac{\ddot{\psi}_p + \ddot{\psi}_{p+1}}{2} \tag{2.4.87}$$

$$\dot{\psi}_{p+1} = \dot{\psi}_p + \frac{\ddot{\psi}_p + \ddot{\psi}_{p+1}}{2} \cdot \Delta t \tag{2.4.88}$$

$$\psi_{p+1} = \psi_p + \dot{\psi}_p \cdot \Delta t + (\ddot{\psi}_p + \ddot{\psi}_{p+1}) \cdot \left(\frac{\Delta t}{2}\right)^2 \tag{2.4.89}$$

which introduced into system 2.4.1, for the *p* interval, it takes the form:

$$\begin{cases} \left[A_{\zeta\zeta p}+B_{\zeta\zeta p}\cdot\frac{\Delta t}{2}+C_{\zeta\zeta p}\cdot\left(\frac{\Delta t}{2}\right)^2\right]\cdot\ddot{\zeta}_{p+1}+\left[A_{\zeta\psi p}+B_{\zeta\psi p}\cdot\frac{\Delta t}{2}+C_{\zeta\psi p}\cdot\left(\frac{\Delta t}{2}\right)^2\right]\cdot\ddot{\psi}_{p+1}= \\ =F_V(\zeta_p,\psi_p,t_{p+1})-F_V(\zeta_p,\psi_p,t_p)+\left[A_{\zeta\zeta p}-B_{\zeta\zeta p}\cdot\frac{\Delta t}{2}-C_{\zeta\zeta p}\cdot\left(\frac{\Delta t}{2}\right)^2\right]\cdot\ddot{\zeta}_p-C_{\zeta\zeta p}\cdot\Delta t\cdot\dot{\zeta}_p+ \\ \left[A_{\zeta\psi p}-B_{\zeta\psi p}\cdot\frac{\Delta t}{2}-C_{\zeta\psi p}\cdot\left(\frac{\Delta t}{2}\right)^2\right]\cdot\ddot{\psi}_p-C_{\zeta\psi p}\cdot\Delta t\cdot\dot{\psi}_p \\ \left[A_{\psi\zeta p}+B_{\psi\zeta p}\cdot\frac{\Delta t}{2}+C_{\psi\zeta p}\cdot\left(\frac{\Delta t}{2}\right)^2\right]\cdot\ddot{\zeta}_{p+1}+\left[A_{\psi\psi p}-B_{\psi\psi p}\cdot\frac{\Delta t}{2}-C_{\psi\psi p}\cdot\left(\frac{\Delta t}{2}\right)^2\right]\cdot\ddot{\psi}_{p+1}= \\ =M_V(\zeta_p,\psi_p,t_{p+1})-M_V(\zeta_p,\psi_p,t_p)+\left[A_{\psi\zeta p}-B_{\psi\zeta p}\cdot\frac{\Delta t}{2}-C_{\psi\zeta p}\cdot\left(\frac{\Delta t}{2}\right)^2\right]\cdot\ddot{\zeta}_p-C_{\psi\zeta p}\cdot\Delta t\cdot\dot{\zeta}_p+ \\ \left[A_{\psi\psi p}-B_{\psi\psi p}\cdot\frac{\Delta t}{2}-C_{\psi\psi p}\cdot\left(\frac{\Delta t}{2}\right)^2\right]\cdot\ddot{\psi}_p-C_{\psi\psi p}\cdot\Delta t\cdot\dot{\psi}_p \end{cases} \tag{2.4.90}$$

where:

$$A_{\zeta\zeta p}=A_{\zeta\zeta}(\zeta_p,\psi_p,t_p) \tag{2.4.91}$$

$$B_{\zeta\zeta p}=B_{\zeta\zeta}(\zeta_p,\psi_p,t_p) \tag{2.4.92}$$

$$C_{\zeta\zeta p}=C_{\zeta\zeta}(\zeta_p,\psi_p,t_p) \tag{2.4.93}$$

$$A_{\zeta\psi p}=A_{\zeta\psi}(\zeta_p,\psi_p,t_p) \tag{2.4.94}$$

$$B_{\zeta\psi p}=B_{\zeta\psi}(\zeta_p,\psi_p,t_p) \tag{2.4.95}$$

$$C_{\zeta\psi p}=C_{\zeta\psi}(\zeta_p,\psi_p,t_p) \tag{2.4.96}$$

$$A_{\psi\zeta p}=A_{\psi\zeta}(\zeta_p,\psi_p,t_p) \tag{2.4.97}$$

$$B_{\psi\zeta p}=B_{\psi\zeta}(\zeta_p,\psi_p,t_p) \tag{2.4.98}$$

$$C_{\psi\zeta p}=C_{\psi\zeta}(\zeta_p,\psi_p,t_p) \tag{2.4.99}$$

$$A_{\psi\psi p}=A_{\psi\psi}(\zeta_p,\psi_p,t_p) \tag{2.4.100}$$

$$B_{\psi\psi p}=B_{\psi\psi}(\zeta_p,\psi_p,t_p) \tag{2.4.101}$$

$$C_{\psi\psi p}=C_{\psi\psi}(\zeta_p,\psi_p,t_p) \tag{2.4.102}$$

The system of 2 non-linear equations with the unknowns $\ddot{\zeta}_{p+1}$ and $\ddot{\psi}_{p+1}$ can be written under the following matrix form:

$$A_p(X_p)\cdot \ddot{X}_{p+1} = E_p(X_p) \tag{2.4.103}$$

where:

$$A_p(X_p) = \begin{bmatrix} A_{p11} & A_{p12} \\ A_{p21} & A_{p22} \end{bmatrix} \tag{2.4.104}$$

$$\begin{aligned} E_p(X_p) &= F_{p+1}(X_p) - F_p(X_p) + \\ &\quad + B_p(X_p)\cdot \ddot{X}_p - C_p(X_p)\cdot \dot{X}_p \end{aligned} \tag{2.4.105}$$

$$F_{p+1}(X_p) = \left\{ F_{Vp+1} \quad M_{Vp+1} \right\}^T \tag{2.4.106}$$

$$F_p(X_p) = \left\{ F_{Vp} \quad M_{Vp} \right\}^T \tag{2.4.107}$$

$$B_p(X_p) = \begin{bmatrix} B_{p11} & B_{p12} \\ B_{p21} & B_{p22} \end{bmatrix} \tag{2.4.108}$$

$$C_p(X_p) = \begin{bmatrix} C_{p11} & C_{p12} \\ C_{p21} & C_{p22} \end{bmatrix} \tag{2.4.109}$$

$$\ddot{X}_{p+1} = \left\{ \ddot{\zeta}_{p+1} \quad \ddot{\psi}_{p+1} \right\}^T \tag{2.4.110}$$

$$\dot{X}_{p+1} = \left\{ \dot{\zeta}_{p+1} \quad \dot{\psi}_{p+1} \right\}^T \tag{2.4.111}$$

$$X_{p+1} = \left\{ \zeta_{p+1} \quad \psi_{p+1} \right\}^T \tag{2.4.112}$$

$$\ddot{X}_p = \left\{ \ddot{\zeta}_p \quad \ddot{\psi}_p \right\}^T \tag{2.4.113}$$

$$\dot{X}_p = \left\{ \dot{\zeta}_p \quad \dot{\psi}_p \right\}^T \tag{2.4.114}$$

$$X_p = \left\{ \zeta_p \quad \psi_p \right\}^T \tag{2.4.115}$$

$$A_{p11} = A_{\zeta\zeta p} + B_{\zeta\zeta p}\cdot\frac{\Delta t}{2} + C_{\zeta\zeta p}\cdot\left(\frac{\Delta t}{2}\right)^2 \tag{2.4.116}$$

$$A_{p12} = A_{\zeta\psi p} + B_{\zeta\psi p}\cdot\frac{\Delta t}{2} + C_{\zeta\psi p}\cdot\left(\frac{\Delta t}{2}\right)^2 \tag{2.4.117}$$

$$A_{p21} = A_{\psi\zeta p} + B_{\psi\zeta p}\cdot\frac{\Delta t}{2} + C_{\psi\zeta p}\cdot\left(\frac{\Delta t}{2}\right)^2 \tag{2.4.118}$$

$$A_{p22} = A_{\psi\psi\, p} + B_{\psi\psi\, p} \cdot \frac{\Delta t}{2} + C_{\psi\psi\, p} \cdot \left(\frac{\Delta t}{2}\right)^2 \tag{2.4.119}$$

$$B_{p11} = A_{\zeta\zeta\, p} - B_{\zeta\zeta\, p} \cdot \frac{\Delta t}{2} - C_{\zeta\zeta\, p} \cdot \left(\frac{\Delta t}{2}\right)^2 \tag{2.4.120}$$

$$B_{p12} = A_{\zeta\psi\, p} - B_{\zeta\psi\, p} \cdot \frac{\Delta t}{2} - C_{\zeta\psi\, p} \cdot \left(\frac{\Delta t}{2}\right)^2 \tag{2.4.121}$$

$$B_{p21} = A_{\psi\zeta\, p} - B_{\psi\zeta\, p} \cdot \frac{\Delta t}{2} - C_{\psi\zeta\, p} \cdot \left(\frac{\Delta t}{2}\right)^2 \tag{2.4.122}$$

$$B_{p22} = A_{\psi\psi\, p} - B_{\psi\psi\, p} \cdot \frac{\Delta t}{2} - C_{\psi\psi\, p} \cdot \left(\frac{\Delta t}{2}\right)^2 \tag{2.4.123}$$

$$C_{p11} = C_{\zeta\zeta\, p} \cdot \Delta t \tag{2.4.124}$$

$$C_{p12} = C_{\zeta\psi\, p} \cdot \Delta t \tag{2.4.125}$$

$$C_{p21} = C_{\psi\zeta\, p} \cdot \Delta t \tag{2.4.126}$$

$$C_{p22} = C_{\psi\psi\, p} \cdot \Delta t \tag{2.4.127}$$

$$F_{Vp} = F_V(\zeta_p, \psi_p, t_p) \tag{2.4.128}$$

$$F_{Vp+1} = F_V(\zeta_p, \psi_p, t_{p+1}) \tag{2.4.129}$$

$$M_{Vp} = M_V(\zeta_p, \psi_p, t_p) \tag{2.4.130}$$

$$M_{Vp+1} = M_V(\zeta_p, \psi_p, t_{p+1}) \tag{2.4.131}$$

The (2.4.103) system solving is made by the Gauss method,

$$\ddot{X}_{p+1} = A_p^{-1}(X_p) \cdot E_p(X_p) \tag{2.4.132}$$

and other motion parameters are matrixally determined with the following relations:

$$\dot{X}_{p+1} = \dot{X}_p + (\ddot{X}_{p+1} + \ddot{X}_p) \cdot \frac{\Delta t}{2} \tag{2.4.133}$$

$$X_{p+1} = X_p + \dot{X}_p \cdot \frac{\Delta t}{2} + (\ddot{X}_{p+1} + \ddot{X}_p) \cdot \left(\frac{\Delta t}{2}\right)^2 \tag{2.4.134}$$

The time integrative method described above goes step by step on each small *p* interval throughout the entire period of oscillation. Because the phenomenon is non-linear, the system matrix and the right member being solution-dependent, to find its stabilized values by applying this method, the iterative solution of the system (2.4.103) by the successive approximation procedure is used in a version adapted to the form of this system.

The algorithm consists in finding of the final solution by successive iterations, starting from a initial solution $X_1^{(0)} = \{0 \quad 0\}^T$, which is used in the calculation of the matrix $A_p(X_p^{(0)})$ and the vector $E_p(X_p^{(0)})$.

Knowing the matrix of the system and its right member, we determine the solution to the first period with the relation:

$$\ddot{X}_{p+1}^{(1)} = A_p^{(0)\text{-}1}(X_p^{(0)}) \cdot E_P(X_p^{(0)}) \tag{2.4.135}$$

Generalizing, the solution at the period *k,* is determined with the relation:

$$\ddot{X}_{p+1}^{(k)} = A_p^{(k-1)-1}(X_p^{(k-1)}) \cdot E_P(X_p^{(k-1)}) \tag{2.4.136}$$

the process continues until the conditions are met at the end of the two successive oscillation period:

$$\left|\zeta_{N_T-1}^{(k)} - \zeta_{N_T}^{(k-1)}\right| \le \varepsilon_\zeta \tag{2.4.137}$$

$$\left|\psi_{N_T-1}^{(k)} - \psi_{N_T}^{(k-1)}\right| \le \varepsilon_\psi \tag{2.4.138}$$

After finding the solution, starting from the relations (2.3.44) and (2.3.50), is calculate the additional shear force and the additional bending moment when the ship moves on waves, on each time interval *p*, along the ship's length.

2.4.3 Program description

Based on the calculation method presented in 2.4.2 has been developed the RLD-V1N program whose code was written in the Visual-FORTRAN language that can be run on 32 or 64-bit computers running Windows XP operating system or a later version.

2.4.4 Verification of the calculation method and of the RLD-V1N program

The verification of the calculation method presented in 2.4.2 and the RLD-V1N program was performed by comparing the results of the calculations with the measurements made on the test model in Mejiro test basin, presented in the paper [70] and described in 2.3.4.

In this case, the verification of the method and the RLD-V1N program was performed by direct analysis of the over time variation of the deck stresses at the middle of the model. These variations are graphically shown in Fig. 2.4.2- 2.4.3 for the three regimes of navigation.

From the analysis of the diagrams shown in these figures, it is found that the results of the calculations are consistent with the measurements, the deviations being generally below 30% and only in isolated cases, such as resonance zones, this limit is exceeded. However, the deviations are like those presented in the literature and accepted as reasonable, so the RLD-V1N method and program can be considered to provide results that can be considered in the industry studies.

2.4.5 Comments and conclusions

The RLD-V1N program for the linear computation of the ship's oscillation parameters and additional sectional efforts induced in its hull by head waves, is a personal achievement and represent a useful tool for design and research activities to improve the construction safety of the ships.

The program was developed based on the classical method presented in Works [58], [59] and [60], using " Ordinary Strip Theory" and "Modified Strip Theory" developed by the author in a specific way to automate calculations, introducing several new considerations:

- determining of the dynamic equilibrium equations on small intervals;
- determining of the additional masses of water, the amortisation and the Smith effect corresponding to these moments, taking into consideration their dependence on ship oscillations and wave parameters, so that the matrix of the linear system of equilibrium equations is no longer symmetrical as it is commonly shown in the literature;
- presenting the complete formulas for the calculation of the wave-induced sectional efforts, which were not found in the literature.
- time calculation and graphical display of the ship’s movements and sectional efforts diagram.

The calculation method presented in 2.4.2 and the RLD-V1N program has been verified on the Mejiro model presented in paper [70] and described in 2.3.4, with acceptable results, so they can be considered validated. It is recommended that, when assessing the sectional efforts, the calculations be made with both variants of the method.

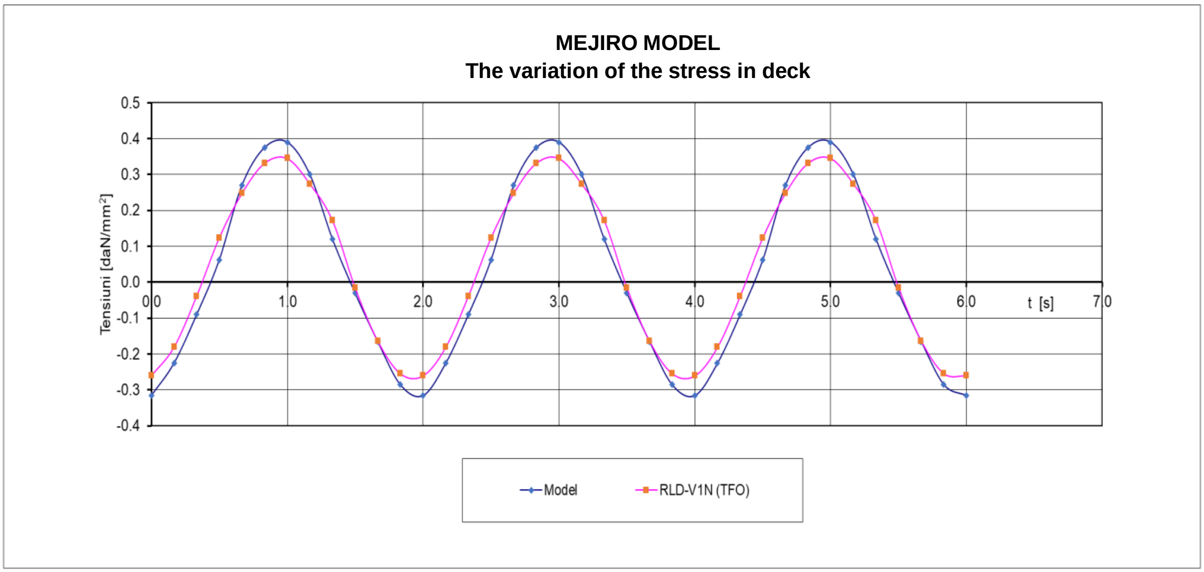


Fig. 2.4.2 – The time variation of the stress in deck, when the model’s speed is 0.
The measurements on the Mejiro model are taken from [70]

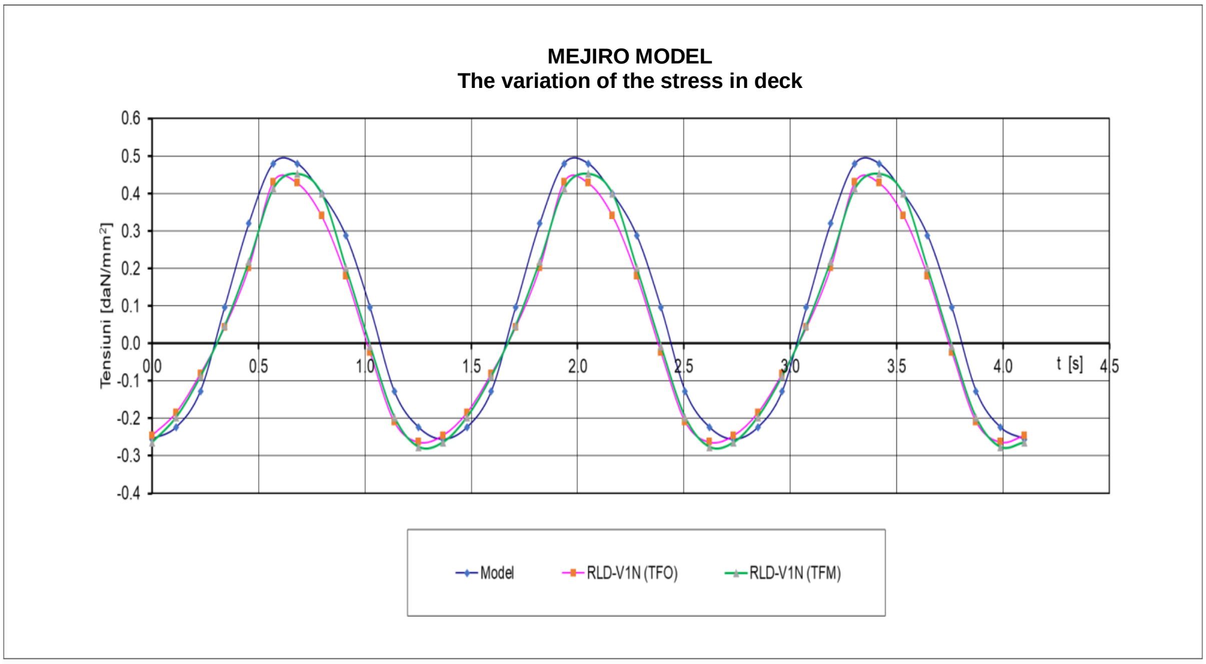


Fig. 2.4.3 – The time variation of the stress in deck, when the model's speed is 1.39 m/s. The measurements on the Mejiro model are taken from [70]

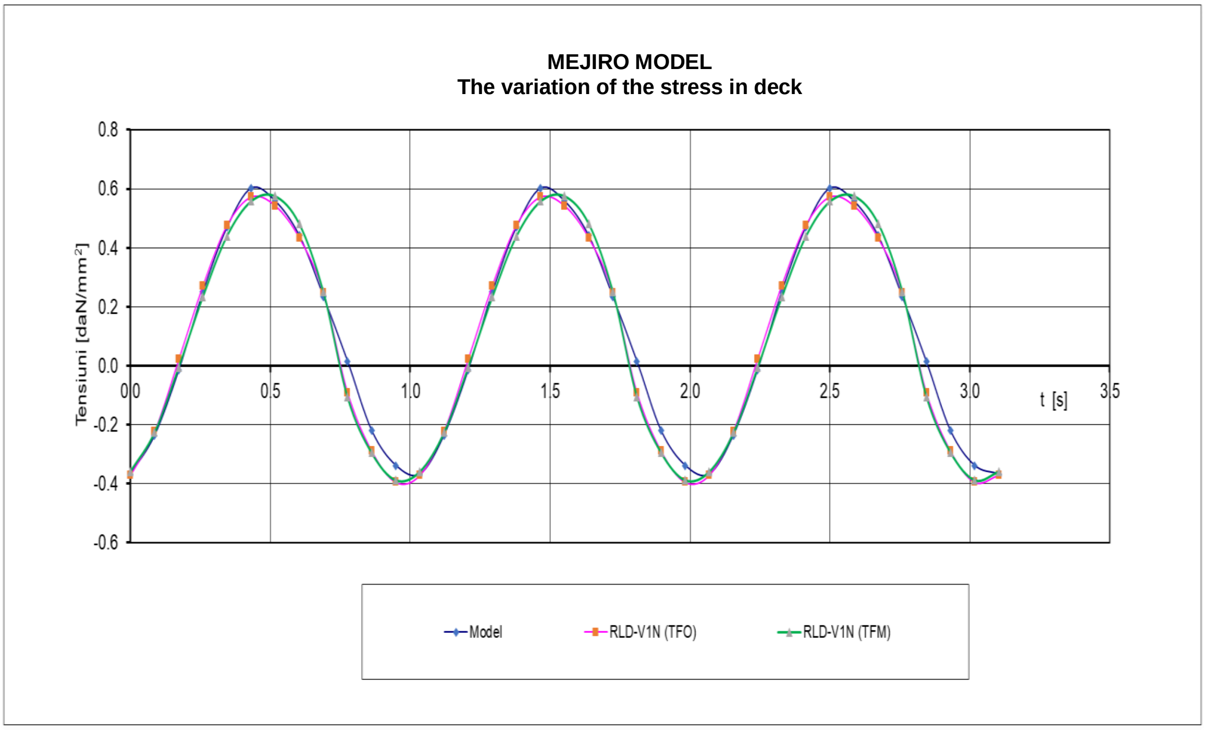


Fig. 2.4.4 – The time variation of the stress in deck, when the model's speed is 2.80 m/s. The measurements on the Mejiro model are taken from [70]

2.5 The program for non-linear calculation of the ship's oscillation parameters and of the additional sectional efforts in its hull induced by head waves, considering quadratic damping.

2.5.1 Program object and destination

The program allows determining of the ship's oscillation parameters as well as the additional sectional efforts in its hull induced by the head wave, considering quadratic damping.

2.5.2 Nonlinear calculation method considering quadratic damping

The theoretical studies and laboratory tests have established that the resistance at movement of a rigid body in a F_R, fluid it is not proportional to the body speed in the fluid, as considered in the methods shown at 2.3.2 and 2.4.2 and in the literature for a convenient calculation, but with the square of the speed, being given by the relation indicated in [71]:

$$F_R = 0.5 \cdot \rho_F \cdot A_C \cdot C_D \cdot V_F \cdot |V_F \cdot| \quad (2.5.1)$$

where:

- ρ_F – fluid density;
- A_C – the area of the projection of the body moving in the fluid on a plane perpendicular to the direction of movement of the fluid;
- C_D – drag coefficient which depends on the shape of the body moving in the fluid. Its values can be found in [72];
- V_F – the relative velocity between fluid and body;

Considering this relation, the formula (2.3.28) of calculating the inertial loads of the distributed mass of the additional water of the element, coupled with those of the hydrodynamic damping, according to the "Ordinary Strip Theory", can be reconsidered as follows:

$$q_a(x,t) = \left\{ \left[\frac{D}{Dt} [M_{33}(x,t) \cdot \frac{Dz_r(x,t)}{Dt} \right] + N_{33}(x,t) \cdot \frac{Dz_r(x,t)}{Dt} \cdot \left| \frac{Dz_r(x,t)}{Dt} \right| \right\} \cdot dx \quad (2.5.2)$$

where product $\frac{Dz_r(x,t)}{Dt} \cdot \left| \frac{Dz_r(x,t)}{Dt} \right|$ can be developed in the form of:

$$\frac{Dz_r(x,t)}{Dt} \cdot \left| \frac{Dz_r(x,t)}{Dt} \right| = [-\dot{\zeta}(t) + x \cdot \dot{\psi}(t) - V \cdot \psi(t) + \frac{h_V}{2} \cdot f_1(x) \cdot \omega \cdot \sin(\frac{2 \cdot \pi \cdot x}{\lambda_V} + \omega_e \cdot t)] \cdot$$
$$| -\dot{\zeta}(t) + x \cdot \dot{\psi}(t) - V \cdot \psi(t) + \frac{h_V}{2} \cdot f_1(x) \cdot \omega \cdot \sin(\frac{2 \cdot \pi \cdot x}{\lambda_V} + \omega_e \cdot t) | \quad (2.5.3)$$

where:

- $M_{33}(x,\zeta,\psi,t)$ – the additional water mass determined in the same way as in the classic method given in 2.3.2, but taking also into account the ship's oscillations;
- $N_{33}(x,\zeta,\psi,t)$ – damping coefficient determined by the relation:

$$N_{33}(x,\zeta,\psi,t) = \rho \cdot y(x,\zeta,\psi,t) \cdot C_D \quad (2.5.4)$$

In addition to these considerations, the distribution of the pressures along the ship length continuously change, as a function which can be considered to vary cosine-like, by following the wave profile or linear or be constantly, on short time intervals. At the same time, the ship's oscillations ζ and ψ also contribute to the change of pressures distributions along the ship length so that the ship's dynamics on the waves is actually complex and non-linear, but it can be mathematically modeled on short time intervals.

For such a short time interval, starting from relations (2.3.51) and (2.3.52), we obtain a system of two differential equations, like the system (2.3.53), having the form:

$$\begin{cases} A_{\zeta\zeta}\cdot\ddot{\zeta}+B_{\zeta\zeta}\cdot\dot{\zeta}+C_{\zeta\zeta}\cdot\zeta+A_{\zeta\psi}\cdot\ddot{\psi}+B_{\zeta\psi}\cdot\dot{\psi}+C_{\zeta\psi}\cdot\psi=F_D(\dot{\zeta},\dot{\psi},\zeta,\psi,t)+F_V(\zeta,\psi,t) \\ A_{\psi\zeta}\cdot\ddot{\zeta}+B_{\psi\zeta}\cdot\dot{\zeta}+C_{\psi\zeta}\cdot\zeta+A_{\psi\psi}\cdot\ddot{\psi}+B_{\psi\psi}\cdot\dot{\psi}+C_{\psi\psi}\cdot\psi=M_D(\dot{\zeta},\dot{\psi},\zeta,\psi,t)+M_V(\zeta,\psi,t) \end{cases} \quad (2.5.5)$$

where, however, the coefficients and free terms are determined by the formulas:

$$A_{\zeta\zeta}(\zeta,\psi,t)=\int_0^L[m(x)+M_{33}(x,\zeta,\psi,t)]\cdot dx \quad (2.5.6)$$

$$B_{\zeta\zeta}(\zeta,\psi,t)=-V\cdot\int_0^L\frac{dM_{33}(x,\zeta,\psi,t)}{dx}\cdot dx \quad (2.5.7)$$

$$C_{\zeta\zeta}(\zeta,\psi,t)=2\cdot\rho\cdot g\cdot\int_0^L y(x,\zeta,\psi,t)\cdot dx \quad (2.5.8)$$

$$A_{\zeta\psi}(\zeta,\psi,t)=-\int_0^L x\cdot[m(x)+M_{33}(x,\zeta,\psi,t)]\cdot dx \quad (2.5.9)$$

$$B_{\zeta\psi}(\zeta,\psi,t)=V\cdot\int_0^L[2\cdot M_{33}(x,\zeta,\psi,t)+x\cdot\frac{dM_{33}(x,\zeta,\psi,t)}{dx}]dx \quad (2.5.10)$$

$$C_{\zeta\psi}(\zeta,\psi,t)=-\int_0^L[2\cdot\rho\cdot g\cdot x\cdot y(x,\zeta,\psi,t)+V^2\cdot\frac{dM_{33}(x,\zeta,\psi,t)}{dx}]\cdot dx \quad (2.5.11)$$

$$A_{\psi\zeta}(\zeta,\psi,t)=\int_0^L x\cdot[m(x)+M_{33}(x,\zeta,\psi,t)]\cdot dx \quad (2.5.12)$$

$$B_{\psi\zeta}(\zeta,\psi,t)=-V\cdot\int_0^L x\cdot\frac{dM_{33}(x,\zeta,\psi,t)}{dx}\cdot dx \quad (2.5.13)$$

$$C_{\psi\zeta}(\zeta,\psi,t)=2\cdot\rho\cdot g\cdot\int_0^L x\cdot y(x,\zeta,\psi,t)\cdot dx \quad (2.5.14)$$

$$A_{\psi\psi}(\zeta,\psi,t)=-\int_0^L x^2\cdot[m(x)+M_{33}(x,t)]\cdot dx \quad (2.5.15)$$

$$B_{\psi\psi}(\zeta,\psi,t)=V\cdot\int_0^L x\cdot[2\cdot M_{33}(x,\zeta,\psi,t)+x\cdot\frac{dM_{33}(x,\zeta,\psi,t)}{dx}]\cdot dx \quad (2.5.16)$$

$$C_{\psi\psi}(\zeta,\psi,t)=-\int_0^L[2\cdot\rho\cdot g\cdot x^2\cdot y(x,\zeta,\psi,t)+x\cdot V^2\cdot\frac{dM_{33}(x,\zeta,\psi,t)}{dx}]dx \quad (2.5.17)$$

$$F_D(\dot{\zeta},\dot{\psi},\zeta,\psi,t)=-\int_0^L N_{33}(x,t)\cdot[\dot{\zeta}(t)-x\cdot\dot{\psi}(t)+V\cdot\psi(t)+\frac{h_V}{2}\cdot f_1(x,\zeta,\psi,t)\cdot\omega\cdot\sin(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)]\cdot$$

$$|\dot{\zeta}(t) - x\cdot\dot{\psi}(t) + V\cdot\psi(t) + \frac{h_V}{2}\cdot f_1(x,\zeta,\psi,t)\cdot\omega\cdot\sin(\frac{2\cdot\pi\cdot x}{\lambda_V} + \omega_e\cdot t)|\cdot dx \qquad (2.5.18)$$

$$F_V(\zeta,\psi,t) = -\omega^2\cdot\frac{h_V}{2}\cdot\int_0^L f_1(x,\zeta,\psi,t)\cdot M_{33}(x,t)\cdot\cos(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)\cdot dx +$$

$$+\omega\cdot V\cdot\frac{h_V}{2}\cdot\int_0^L f_1(x,\zeta,\psi,t)\cdot\frac{dM_{33}(x,t)}{dx}\cdot\sin(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)\cdot dx +$$

$$+\rho\cdot g\cdot h_V\cdot\int_0^L f_1(x,\zeta,\psi,t)\cdot y(x)\cdot\cos(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)\cdot dx \qquad (2.5.19)$$

$$M_D(\dot{\zeta},\dot{\psi},\zeta,\psi,t) = -\int_0^L x\cdot N_{33}(x,t)\cdot[\dot{\zeta}(t) - x\cdot\dot{\psi}(t) + V\cdot\psi(t) + \frac{h_V}{2}\cdot f_1(x,\zeta,\psi,t)\cdot\omega\cdot\sin(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)]\cdot$$

$$|\dot{\zeta}(t) - x\cdot\dot{\psi}(t) + V\cdot\psi(t) + \frac{h_V}{2}\cdot f_1(x,\zeta,\psi,t)\cdot\omega\cdot\sin(\frac{2\cdot\pi\cdot x}{\lambda_V} + \omega_e\cdot t)|\cdot dx \qquad (2.5.20)$$

$$M_V(\zeta,\psi,t) = -\omega^2\cdot\frac{h_V}{2}\cdot\int_0^L f_1(x,\zeta,\psi,t)\cdot x\cdot M_{33}(x,t)\cdot\cos(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)\cdot dx +$$

$$+\omega\cdot V\cdot\frac{h_V}{2}\cdot\int_0^L f_1(x,\zeta,\psi,t)\cdot x\cdot\frac{dM_{33}(x,t)}{dx}\cdot\sin(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)\cdot dx +$$

$$+\rho\cdot g\cdot h_V\cdot\int_0^L f_1(x,\zeta,\psi,t)\cdot x\cdot y(x)\cdot\cos(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)\cdot dx \qquad (2.5.21)$$

$f_1(x,\zeta,\psi,t)$ - is determined by the relation (2.4.16).

Although the system of the differential equations of order two (2.5.5) is nonlinear, it can be assumed that, on small time intervals, the additional masses, the damping and the Smith effect are linear, so that this system is also linear at such intervals, and its stabilized solution varies according to a cosinusoidal rule as those of the external loads, so it can be admitted that it is given by the relations (2.3.72) and (2.3.73).

By replacing these solutions in (2.5.5) and grouping by cos and sin, we obtain the system of two equations in which the unknowns are ζ_1, ζ_2, ψ_1, ψ_2:

$$\begin{cases} [(C_{\zeta\zeta} - \omega_e^2 A_{\zeta\zeta})\cdot\zeta_1 + \omega_e\cdot B_{\zeta\zeta}\cdot\zeta_2 + (C_{\zeta\psi} - \omega_e^2 A_{\zeta\psi})\cdot\psi_1 + \omega_e\cdot B_{\zeta\psi}\cdot\psi_2]\cdot\cos(\omega_e\cdot t) + \\ +[-\omega_e\cdot B_{\zeta\zeta}\cdot\zeta_1 + (C_{\zeta\zeta} - \omega_e^2 A_{\zeta\zeta})\cdot\zeta_2 - \omega_e\cdot B_{\zeta\psi}\cdot\psi_1 + (C_{\zeta\psi} - \omega_e^2 A_{\zeta\psi})\cdot\psi_2]\cdot\sin(\omega_e\cdot t) = \\ = F_D(\zeta_1,\zeta_2,\psi_1,\psi_2,t) + F_V(\zeta_1,\zeta_2,\psi_1,\psi_2,t) \\ \\ [(C_{\psi\zeta} - \omega_e^2 A_{\psi\zeta})\cdot\zeta_1 + \omega_e\cdot B_{\psi\zeta}\cdot\zeta_2 + (C_{\psi\psi} - \omega_e^2 A_{\psi\psi})\cdot\psi_1 + \omega_e\cdot B_{\psi\psi}\cdot\psi_2]\cdot\cos(\omega_e\cdot t) + \\ +[-\omega_e\cdot B_{\psi\zeta}\cdot\zeta_1 + (C_{\psi\zeta 1} - \omega_e^2 A_{\psi\zeta 1})\cdot\zeta_2 + (C_{\psi\psi 1} - \omega_e^2 A_{\xi\psi 1})\cdot\psi_2 - \omega_e\cdot B_{\psi\psi}\cdot\psi_1]\cdot\sin(\omega_e\cdot t) = \\ = M_D(\zeta_1,\zeta_2,\psi_1,\psi_2,t) + M_V(\zeta_1,\zeta_2,\psi_1,\psi_2,t) \end{cases} \qquad (2.5.22)$$

where:

$$F_D(\zeta_1,\zeta_2,\psi_1,\psi_2,t)=-\int_0^L N_{33}(x)\cdot\{\omega_e\cdot[-\zeta_1\cdot\sin(\omega_e\cdot t)+\zeta_2\cdot\cos(\omega_e\cdot t)]-$$
$$-\omega_e\cdot x\cdot[-\psi_1\cdot\sin(\omega_e\cdot t)+\psi_2\cdot\cos(\omega_e\cdot t)]+V\cdot[\psi_1\cdot\cos(\omega_e\cdot t)+\psi_2\cdot\sin(\omega_e\cdot t)]+$$
$$+\frac{h_V}{2}\cdot f_1(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)\cdot\omega\cdot\sin(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)\}\cdot \tag{2.5.23}$$
$$\cdot|\,\omega_e\cdot[-\zeta_1\cdot\sin(\omega_e\cdot t)+\zeta_2\cdot\cos(\omega_e\cdot t)]-\omega_e\cdot x\cdot[-\psi_1\cdot\sin(\omega_e\cdot t)+\psi_2\cdot\cos(\omega_e\cdot t)]+$$
$$+V\cdot[\psi_1\cdot\cos(\omega_e\cdot t)+\psi_2\cdot\sin(\omega_e\cdot t)]+\frac{h_V}{2}\cdot f_1(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)\cdot\omega\cdot\sin(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)\,|\cdot dx$$

$$F_V(\zeta_1,\zeta_2,\psi_1,\psi_2,t)=-\omega^2\cdot\frac{h_V}{2}\cdot\int_0^L f_1(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)\cdot M_{33}(x)\cdot\cos(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)\cdot dx+$$
$$+\omega\cdot V\cdot\frac{h_V}{2}\cdot\int_0^L f_1(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)\cdot\frac{dM_{33}(x)}{dx}\cdot\sin(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)\cdot dx+$$
$$+\rho\cdot g\cdot h_V\cdot\int_0^L f_1(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)\cdot y(x)\cdot\cos(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)\cdot dx \tag{2.5.24}$$

$$M_D(\zeta_1,\zeta_2,\psi_1,\psi_2,t)=-\int_0^L x\cdot N_{33}(x)\cdot\{\omega_e\cdot[-\zeta_1\cdot\sin(\omega_e\cdot t)+\zeta_2\cdot\cos(\omega_e\cdot t)]-$$
$$-\omega_e\cdot x\cdot[-\psi_1\cdot\sin(\omega_e\cdot t)+\psi_2\cdot\cos(\omega_e\cdot t)]+V\cdot[\psi_1\cdot\cos(\omega_e\cdot t)+\psi_2\cdot\sin(\omega_e\cdot t)]+$$
$$+\frac{h_V}{2}\cdot f_1(x,t)\cdot\omega\cdot\sin(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)\}\cdot \tag{2.5.25}$$
$$\cdot|\,\omega_e\cdot[-\zeta_1\cdot\sin(\omega_e\cdot t)+\zeta_2\cdot\cos(\omega_e\cdot t)]-\omega_e\cdot x\cdot[-\psi_1\cdot\sin(\omega_e\cdot t)+\psi_2\cdot\cos(\omega_e\cdot t)]+$$
$$+V\cdot[\psi_1\cdot\cos(\omega_e\cdot t)+\psi_2\cdot\sin(\omega_e\cdot t)]+\frac{h_V}{2}\cdot f_1(x,t)\cdot\omega\cdot\sin(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)\,|\cdot dx\cdot$$

$$M_V(\zeta_1,\zeta_2,\psi_1,\psi_2,t)=-\omega^2\cdot\frac{h_V}{2}\cdot\int_0^L f_1(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)\cdot x\cdot M_{33}(x)\cdot\cos(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)\cdot dx-$$
$$+\omega\cdot V\cdot\frac{h_V}{2}\cdot\int_0^L f_1(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)\cdot x\cdot\frac{dM_{33}(x)}{dx}\cdot\sin(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)\cdot dx+$$
$$+\rho\cdot g\cdot h_V\cdot\int_0^L f_1(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)\cdot x\cdot y(x)\cdot\cos(\frac{2\cdot\pi\cdot x}{\lambda_V}+\omega_e\cdot t)\cdot dx \tag{2.5.26}$$

$f_1(x,\zeta_1,\zeta_2,\psi_1,\psi_2,t)$ - is determined by relation (2.4.22).

Because there are only two equations, the 4 unknowns ζ_1, ζ_2, ψ_1, ψ_2, are determined by balancing the ship at the limits of the time intervals, defined by the relations:

$$t_p = \frac{2\cdot\pi}{\omega_e \cdot N_T}\cdot p$$
$$t_{p+1} = \frac{2\cdot\pi}{\omega_e \cdot N_T}\cdot (p+1) \tag{2.5.27}$$

for each interval *p*, resulting a system of 4 non-linear equations, with the unknowns ζ_{p1}, ζ_{p2}, ψ_{p1}, ψ_{p2}, which can be written under the following matrix form:

$$A_p(X_p)\cdot X_p = F_p(X_p) \tag{2.5.28}$$

where :

$$A_p(X_p) = \begin{bmatrix} A_{p11} & A_{p12} & A_{p13} & A_{p14} \\ A_{p21} & A_{p22} & A_{p23} & A_{p24} \\ A_{p31} & A_{p32} & A_{p33} & A_{p34} \\ A_{p41} & A_{p42} & A_{p43} & A_{p44} \end{bmatrix} \tag{2.5.29}$$

$$X_p = \{\zeta_{p1} \quad \zeta_{p2} \quad \psi_{p1} \quad \psi_{p2}\}^T \tag{2.5.30}$$

$$F_p(X_p) = F_{Dp}(X_p) + F_{Vp}(X_p) \tag{2.5.31}$$

$$F_{Dp}(X_p) = \{F_{Dp1}(X_p) \quad F_{Dp2}(X_p) \quad M_{Dp1}(X_p) \quad M_{Dp2}(X_p)\}^T \tag{2.5.32}$$

$$F_{Vp}(X_p) = \{F_{Vp1}(X_p) \quad F_{Vp2}(X_p) \quad M_{Vp1}(X_p) \quad M_{Vp2}(X_p)\}^T \tag{2.5.33}$$

$$F_{Dp1}(X_p) = F_D(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_p) \tag{2.5.34}$$

$$F_{Dp2}(X_p) = F_D(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_{p+1}) \tag{2.5.35}$$

$$M_{Dp1}(X_p) = M_D(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_p) \tag{2.5.36}$$

$$M_{Dp2}(X_p) = M_D(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_{p+1}) \tag{2.5.37}$$

$$F_{Vp1}(X_p) = F_V(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_p) \tag{2.5.38}$$

$$F_{Vp2}(X_p) = F_V(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_{p+1}) \tag{2.5.39}$$

$$M_{Vp1}(X_p) = M_V(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_p) \tag{2.5.40}$$

$$M_{Vp2}(X_p) = M_V(\zeta_{p1}, \zeta_{p2}, \psi_{p1}, \psi_{p2}, t_{p+1}) \tag{2.5.41}$$

The coefficients of the matrix $A_p(X_p)$, are determined by relations 2.4.30 – 2.4.45.

The non-linear system solving (2.5.28) is performed by the successive approximation method, in a version adapted to the shape of this system, similar to the solving of the system (2.4.26).

The successive approximation method has the disadvantage that in the resonance zones it becomes divergent, and in this case shall be used the β-Newmark time integration method presented in [60] and [73], adapted and developed to solve the (2.5.1) system.

According to this method, the oscillation period is divided in N_T sufficiently short intervals of Δt duration, delimited by the times defined by (2.5.27), on which the acceleration is considered to be constant and equal to the arithmetic mean of the values from the ends of these intervals, so there are relations (2.4.84) - (2.4.89), which introduced into system 2.5.1, for the *p* interval, it takes the form:

$$\begin{cases} \left[A_{\zeta\zeta p}+B_{\zeta\zeta p}\cdot\frac{\Delta t}{2}+C_{\zeta\zeta p}\cdot\left(\frac{\Delta t}{2}\right)^2\right]\cdot\ddot{\zeta}_{p+1}+\left[A_{\zeta\psi p}+B_{\zeta\psi p}\cdot\frac{\Delta t}{2}+C_{\zeta\psi p}\cdot\left(\frac{\Delta t}{2}\right)^2\right]\cdot\ddot{\psi}_{p+1}= \\ =F_D(\dot{\zeta}_p,\dot{\psi}_p,\zeta_p,\psi_p,t_{p+1})-F_D(\dot{\zeta}_p,\dot{\psi}_p,\zeta_p,\psi_p,t_p)+ \\ +F_V(\zeta_p,\psi_p,t_{p+1})-F_V(\zeta_p,\psi_p,t_p)+\left[A_{\zeta\zeta p}-B_{\zeta\zeta p}\cdot\frac{\Delta t}{2}-C_{\zeta\zeta p}\cdot\left(\frac{\Delta t}{2}\right)^2\right]\cdot\ddot{\zeta}_p-C_{\zeta\zeta p}\cdot\Delta t\cdot\dot{\zeta}_p+ \\ \left[A_{\zeta\psi p}-B_{\zeta\psi p}\cdot\frac{\Delta t}{2}-C_{\zeta\psi p}\cdot\left(\frac{\Delta t}{2}\right)^2\right]\cdot\ddot{\psi}_p-C_{\zeta\psi p}\cdot\Delta t\cdot\dot{\psi}_p \\ \left[A_{\psi\zeta p}+B_{\psi\zeta p}\cdot\frac{\Delta t}{2}+C_{\psi\zeta p}\cdot\left(\frac{\Delta t}{2}\right)^2\right]\cdot\ddot{\zeta}_{p+1}+\left[A_{\psi\psi p}-B_{\psi\psi p}\cdot\frac{\Delta t}{2}-C_{\psi\psi p}\cdot\left(\frac{\Delta t}{2}\right)^2\right]\cdot\ddot{\psi}_{p+1}= \\ =M_D(\dot{\zeta}_p,\dot{\psi}_p,\zeta_p,\psi_p,t_{p+1})-M_D(\dot{\zeta}_p,\dot{\psi}_p,\zeta_p,\psi_p,t_p)+ \\ +M_V(\zeta_p,\psi_p,t_{p+1})-M_V(\zeta_p,\psi_p,t_p)+\left[A_{\psi\zeta p}-B_{\psi\zeta p}\cdot\frac{\Delta t}{2}-C_{\psi\zeta p}\cdot\left(\frac{\Delta t}{2}\right)^2\right]\cdot\ddot{\zeta}_p-C_{\psi\zeta p}\cdot\Delta t\cdot\dot{\zeta}_p+ \\ \left[A_{\psi\psi p}-B_{\psi\psi p}\cdot\frac{\Delta t}{2}-C_{\psi\psi p}\cdot\left(\frac{\Delta t}{2}\right)^2\right]\cdot\ddot{\psi}_p-C_{\psi\psi p}\cdot\Delta t\cdot\dot{\psi}_p \end{cases} \quad (2.5.42)$$

where the notations from (2.4.91) - (2.4.102) were used:

The system of 2 non-linear equations with the unknowns $\ddot{\zeta}_{p+1}$ and $\ddot{\psi}_{p+1}$ can be written under the following matrix form:

$$A_p(X_p)\cdot\ddot{X}_{p+1}=E_p(X_p) \quad (2.5.43)$$

and is resolved similarly to the system (2.4.103) with the difference that:

$$F_{p+1}(X_p)=\left\{F_{Dp+1}+F_{Vp+1} \quad M_{Dp+1}+M_{Vp+1}\right\}^T \quad (2.5.44)$$

$$F_p(X_p)=\left\{F_{Dp}+F_{Vp} \quad M_{Dp}+M_{Vp}\right\}^T \quad (2.5.45)$$

$$F_{Dp}=F_D(\dot{\zeta}_p,\dot{\psi}_p\zeta_p,\psi_p,t_p) \quad (2.5.46)$$

$$F_{Dp+1}=F_D(\dot{\zeta}_p,\dot{\psi}_p,\zeta_p,\psi_p,t_{p+1}) \quad (2.5.47)$$

$$F_{Vp}=F_V(\zeta_p,\psi_p,t_p) \quad (2.5.48)$$

$$F_{Vp+1}=F_V(\zeta_p,\psi_p,t_{p+1}) \quad (2.5.49)$$

$$M_{Dp}=M_D(\dot{\zeta}_p,\dot{\psi}_p\zeta_p,\psi_p,t_p) \quad (2.5.50)$$

$$M_{Dp+1}=M_D(\dot{\zeta}_p,\dot{\psi}_p,\zeta_p,\psi_p,t_{p+1}) \quad (2.5.52)$$

$$M_{Vp}=M_V(\zeta_p,\psi_p,t_p) \quad (2.5.53)$$

$$M_{Vp+1}=M_V(\zeta_p,\psi_p,t_{p+1}) \quad (2.5.54)$$

After finding the solution, starting from the relations (2.3.44) and (2.3.50), is calculate the additional shear force and the additional bending moment when the ship moves on waves, on each time interval *p*, along the ship's length.

2.5.3 Program description

Based on the calculation method presented in 2.5.2 has been developed the RLD-V2 program whose code was written in the Visual-FORTRAN language that can be run on 32 or 64-bit computers running Windows XP operating system or a later version.

2.5.4 Verification of the calculation method and of the RLD-V2 program

The verification of the calculation method presented in 2.5.2 and the RLD-V2 program was performed by comparing the results of the calculations with the measurements made on the test model in Mejiro test basin, presented in the paper [70] and described in 2.3.4.

Also in this case, the verification of the method and the RLD-V2 program was performed by direct analysis of the over time variation of the deck stress at the middle of the model. These variations are graphically shown in Fig. 2.5.1- 2.5.3 for the three regimes of navigation.

From the analysis of the diagrams shown in these figures, it is found that the results of the calculations are consistent with the measurements on the model, the deviations being generally below 30% and only in isolated cases, such as resonance zones, this limit is exceeded. As in the case of other models, these deviations are justified by the complexity of the hydrodynamics of the ship on waves, in which alongside the ship also participate additional masses of water difficult to accurately estimate, and the damping phenomena are equally difficult to determine with precision. Also, the weight distribution of the Mejiro model was adopted without sufficient data. However, the deviations are similar to those presented in the literature and accepted as reasonable.

The above calculations show that for the Mejiro model, the method outlined in 2.5.2 allows for near-measurement results. Greater approximation of the results calculations with the measurement would be possible through a better appreciation of the additional masses and damping.

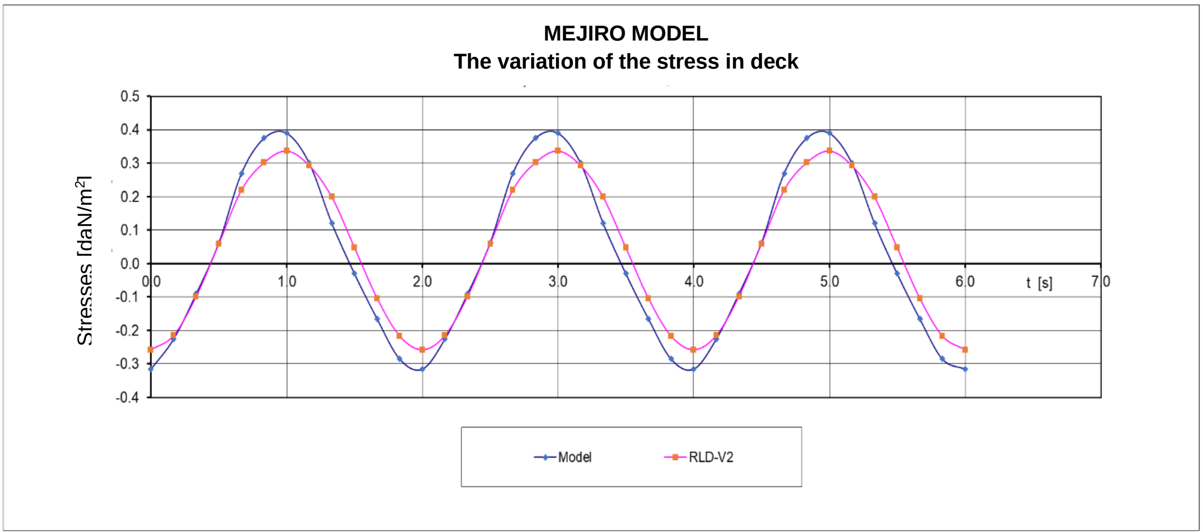


Fig. 2.5.1 – The time variation of the stress in deck, when the model’s speed is 0. The measurements on the Mejiro model are taken from [70]

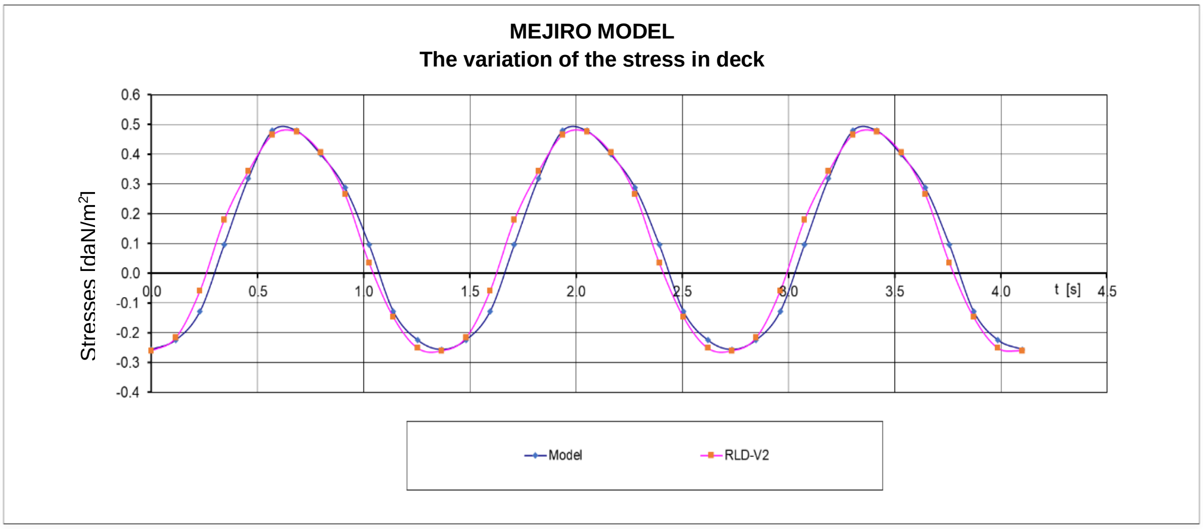


Fig. 2.5.2 – The time variation of the stress in deck, when the model's speed is 1.39 m/s.. The measurements on the Mejiro model are taken from [70]

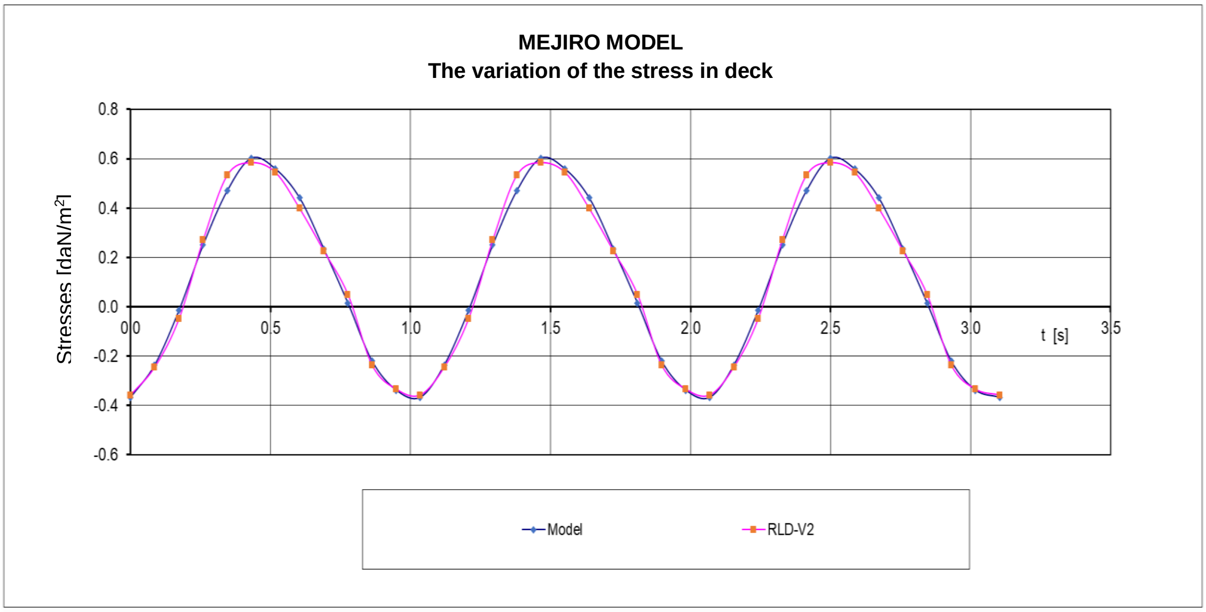


Fig. 2.5.3 – The time variation of the stress in deck, when the model's speed is 2.80 m/s. The measurements on the Mejiro model are taken from [70]

2.5.5 Comments and conclusions

The RLD-V2 program was developed based on the method presented in 2.5.2, based on Morisson equation where the damping is considered to vary with the square of the speed, and developed by the author in a specific way to automate calculations, introducing several new considerations:

- determining of the dynamic equilibrium equations on short intervals where the damping is considered to vary with the square of the speed;
- determining of the additional masses of water, the amortisation and the Smith effect corresponding to these moments, taking into consideration their dependence on ship oscillations and wave parameters, so that the matrix of the linear system of

equilibrium equations is no longer symmetrical as it is commonly shown in the literature;

- time calculation and graphical display of the ship's movements and sectional efforts diagram.

The calculation method presented in 2.5.2 and the RLD-V2 program has been verified on the Mejiro model presented in paper [70] and described in 2.3.4, noting that the results of the calculations are consistent with the measurements, the deviations being generally below 30% and only in isolated cases, such as resonance zones, this limit is exceeded. However, the deviations are like those presented in the literature and accepted as reasonable.

In Fig. 2.5.4 a comparison of the deck stresses calculated by this program with those obtained with the RLD-V1 and RLD-V1N programs, as well as measurements from the Mejiro model, are made. It is found that the deviations between the results obtained with the three programs are small, with the indication that the results of the RLD-V2 program are closer to the measurements, but with the observation that in the resonance zone at the speed of 1.39 m/s, the results of the RLD-V1N program, the TFM variant, are slightly closer to the measurements.

The calculations presented in Table 2.5.2 indicate that up to F_n = 0.20, it is sufficient to verify the longitudinal strength of the ship to the action of the waves, only by quasi-static laying, which is simpler to apply.

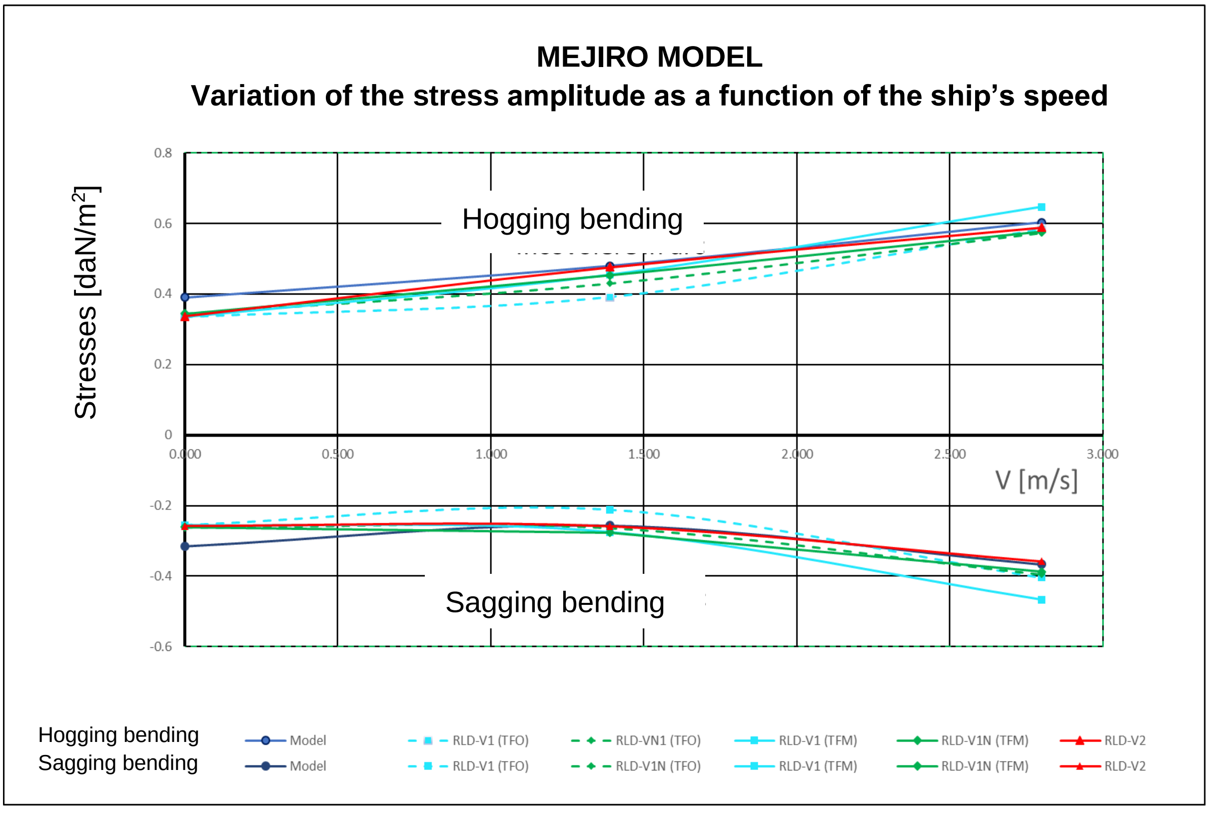


Fig. 2.5.4 – The variation of the deck stresses as a function of the speed in hogging or sagging condition.
The measurements on the Mejiro model are taken from [70]

3 IMPROVING THE CONSTRUCTION SAFETY OF THE SHIPS REGARDING LONGITUDINAL STRENGTH IN INTACT CONDITION

3.1 Overview

In order to make improvements to the existing regulations on longitudinal strength of the intact ships, aligned with the IACS method [74], an analysis of these was made using the tools presented in the chapter 2 and where it has been found that they can be completed or can be improved, proposals substantiated for this purpose have been made.

3.2 Presentation of the IACS method

In order to determining of the sectional efforts due to the waves occurring in the hull of the seagoing ships, the classification societies have aligned their calculation methods with those established by the IACS [74.S11].

According to this method, for seagoing ships other than container ships, bulk carriers and oil tankers with double hull, the bending moment induced by the wave which occurs with a 10^{-8} probability, for navigation at sea in the wave propagation direction, is given by the following formulas:

- for hogging bending:

$$M_{WV,H} = \kappa_H \cdot F_M \cdot C \cdot L^2 \cdot B \cdot C_B \cdot 10^{-3} \text{ [kN m]} \qquad (3.2.1.)$$

- for sagging bending

$$M_{WV,S} = k_S \cdot F_M \cdot C \cdot L^2 \cdot B \cdot (C_B + 0.7) \cdot 10^{-3} \text{ [kN m]} \qquad (3.2.2)$$

where:

L – Length of the ship [m];
B – Breadth of the ship [m];
$k_H = 190$ – Hogging bending coefficient;
$k_S = 110$ – sagging bending coefficient;
F_M – distribution factor defined in table 3.2.1;
C_B – Ship's block coefficient for draught at full loading draught;
C – The wave parameter (represents the height of the wave corrected due to the Smith effect) determined with the relations:

$$C = \left(\frac{L}{25} + 4.1\right) \qquad \text{for L ≤ 90 m}$$

$$C = \left[10.75 - \left(\frac{300 - L}{100}\right)^{1.5}\right] \qquad \text{for 90 ≤ L ≤ 300 m} \qquad (3.2.3)$$

$$C = 10.75 \qquad \text{for 300 ≤ L ≤ 350 m}$$

$$C = \left[10.75 - \left(\frac{L - 350}{100}\right)^{1.5}\right] \qquad \text{for 350 ≤ L ≤ 500 m}$$

Table 3.2.1 – Distribution factor F_M

Position of the transverse section of the hull	**Distribution factor F_M**
$0 \le x < 0.4 \cdot L$	$2.5 \cdot \frac{x}{L}$
$0.4 \cdot L \le x \le 0.65 \cdot L$	1
$0.65 \cdot L < x \le L$	$2.86 \cdot \left(1 - \frac{x}{L}\right)$

The wave-induced vertical shear force appearing in a transverse section of the hull, for heavy sea navigation, parallel to the wave propagation direction, is determined according to the IACS method, for the same category of vessel, with the formula:

$$Q_{WV} = k_Q \cdot F_Q \cdot C \cdot L \cdot B \cdot (C_B + 0.7) \cdot 10^{-2} \text{ [kN]} \tag{3.2.4}$$

where :

$k_Q = 30$ – shear coefficient;

F_Q – distribution factor defined in table 3.2.2 for positive and negative shear forces;

Tabel 3.2.2 – Distribution factor F_Q

Position of the section	**Distribution factor F_Q**	
	Positive shear force	**Negative shear force**
$0 \le x < 0.2 \cdot L$	$4.6 \cdot A \cdot \frac{x}{L}$	$4.6 \cdot \frac{x}{L}$
$0.2 \cdot L \le x \le 0.3 \cdot L$	$0.92 \cdot A$	0.92
$0.3 \cdot L < x < 0.4 \cdot L$	$(0.92 \cdot A - 7) \cdot \left(0.4 - \frac{x}{L}\right) + 0.7$	$2.2 \cdot \left(0.4 - \frac{x}{L}\right) + 0.7$
$0.4 \cdot L \le x \le 0.6 \cdot L$	0.7	0.7
$0.6 \cdot L < x < 0.7 \cdot L$	$3 \cdot \left(\frac{x}{L} - 0.6\right) + 0.7$	$(10 \cdot A - 7) \cdot \left(\frac{x}{L} - 0.6\right) + 0.7$
$0.7 \cdot L \le x \le 0.85 \cdot L$	1	A
$0.85 \cdot L < x \le L$	$6.67 \cdot \left(1 - \frac{x}{L}\right)$	$6.67 \cdot A \cdot \left(1 - \frac{x}{L}\right)$
Note: $A = \frac{190 \cdot C_B}{110 \cdot (C_B + 0.7)}$		

For containerships, IACS proposed the method in [74.S11A] and for double-hull bulk carriers and tanks, the method from [5.Ch4.Sec4.3], which are similar to the one described above, and close results are obtained.

3.3 Verification of the IACS method for determining of the sectional efforts in the intact seagoing ship’s hull induced by the wave, based on the method of the quasi-static layout of the ship on the wave.

The verification of the wave-induced sectional stresses determined according to IACS, can be done through direct calculations and various methods have been adopted based on assumptions that reduce the complexity of calculations without significantly affecting the accuracy of the results of the calculations against to actual values.

A first direct and efficient method consists of the quasi-static layout of the ship on the wave, as described in 2.2.

The method ensures accurate results for navigation on stern wave parallel to the direction of movement of the ship.

At positioning on quasi-static wave, it is necessary to take into account the Smith effect with approx. 15% reduction of the hydrostatic pressure by depth, as a result of the orbital motion of the wave particles.

For the determination of additional sectional efforts induced by a quasi-static wave along the ship length, the RLS-V1 program described in 2.2 was used by applying the formulas:

- for additional shear force:

$$Q_{WV}(x) = Q_{TW}(x) - Q_{SW}(x) \tag{3.3.1}$$

- for additional bending moment along the ship length:

$$M_{WV}(x) = M_{TW}(x) - M_{SW}(x) \tag{3.3.2}$$

where:

$Q_{TW}(x)$ – the total shear force at the quasi-static layout on the wave;
$Q_{SW}(x)$ – the shear force at the layout on still water [kN];
$M_{TW}(x)$ – the total bending moment at the quasi-static layout on the wave;
$M_{SW}(x)$ – the bending moment at the layout on still water [kNm].

3.3.1 Verification of the IACS method for a 15,000 tdw general cargo ship

The methodology for calculating the additional sectional efforts, induced by the quasi-static wave into the ship's hull, was applied to a 15,000 tdw general cargo ship presented in Figure 3.3.1, to verify the IACS method.

The main characteristics of the ship are indicated below:

Lmax = 162.30 m
L = 155.00 m
B = 22.20 m
D = 13.40m
T = 10.10m

This one was placed quasi-statically on a wave with the height corrected by the Smith effect, equal to the value C determined by the formula (3.2.3), i.e. equal to 8.997 m and a length equal to the length of the ship (the real wave having a height of 11.000 m, the period of 9.96 s, the length of 155 m and the speed of 15.6 m/s, occurs with a probability of 0.04%, as indicated by the statistical measurements presented in [75]).

The results of the calculations are shown in Table 3.3.1 and graphically in Figures 3.3.2 and 3.3.3.

In the same table and graph are gived also the additional wave-induced efforts according to the IACS method [74.S11].

Table 3.3.1 – Comparative results of the calculations of the additional sectional efforts induced by waves for the 15000 tdw cargo ship

Equilibrium parameters, additional efforts induced by waves	**Quasi-static layout on the cosine wave**		**Quasi-static layout on the trochoidal wave**		**IACS Method**		**Differences in [%] between layout on the cosine wave and trochoidal wave**		**Differences in [%] between IACS method and quasi-static layout on the trochoidal wave**	
	On the crest	On the hollow	On the crest	On the hollow	Hogging bending	Sagging bending	On the crest	On the hollow	Hogging/ On the crest	Sagging/ On the hollow
T_o *[m]*	7.599	12.630	8.119	13.117	-	-	-6.405	-3.713	-	-
ψ [rad]	0.658	-0.297	0.677	-0.217	-	-	-2.806	36.866	-	-
Θ [rad]	0	0	0	0	0	0	0.000	0.000	0.000	0.000
$Q_{WV}(+)$*[kN]*	14622	18003	15164	17587	12353	12353	-3.574	2.365	-22.756	-42.370
$Q_{WV}(-)$*[kN]*	-14115	-18943	-14314	-17111	-11365	-11365	-1.390	10.707	-25.948	-50.559
M_{WV} *[kNm]*	589390	-838078	599373	-797591	574646	-700266	-1.666	5.076	-4.303	-13.898

It is found that the maximum additional bending moments determined by static laying of the ship on wave are up to 14% higher than those determined by IACS, and in the case of shear forces, the differences are much higher, reaching up to 51%, which means that the relations indicated by IACS lead to sub-dimensioning of ships in terms of wave-induced sectional efforts.

It is also found that the differences between the maximum sectional efforts when the ship is placed on a cosine and trohoidal wave are below 10.7%, which allows the approximation of the actual trohoidal wave with a cosine wave to perform the analysis regarding behavior of the ship on waves with less effort but keeping the accuracy of the results within acceptable limits.

Fig. 3.3.1 – The cargo ship of 15000 tdw that was analyzed

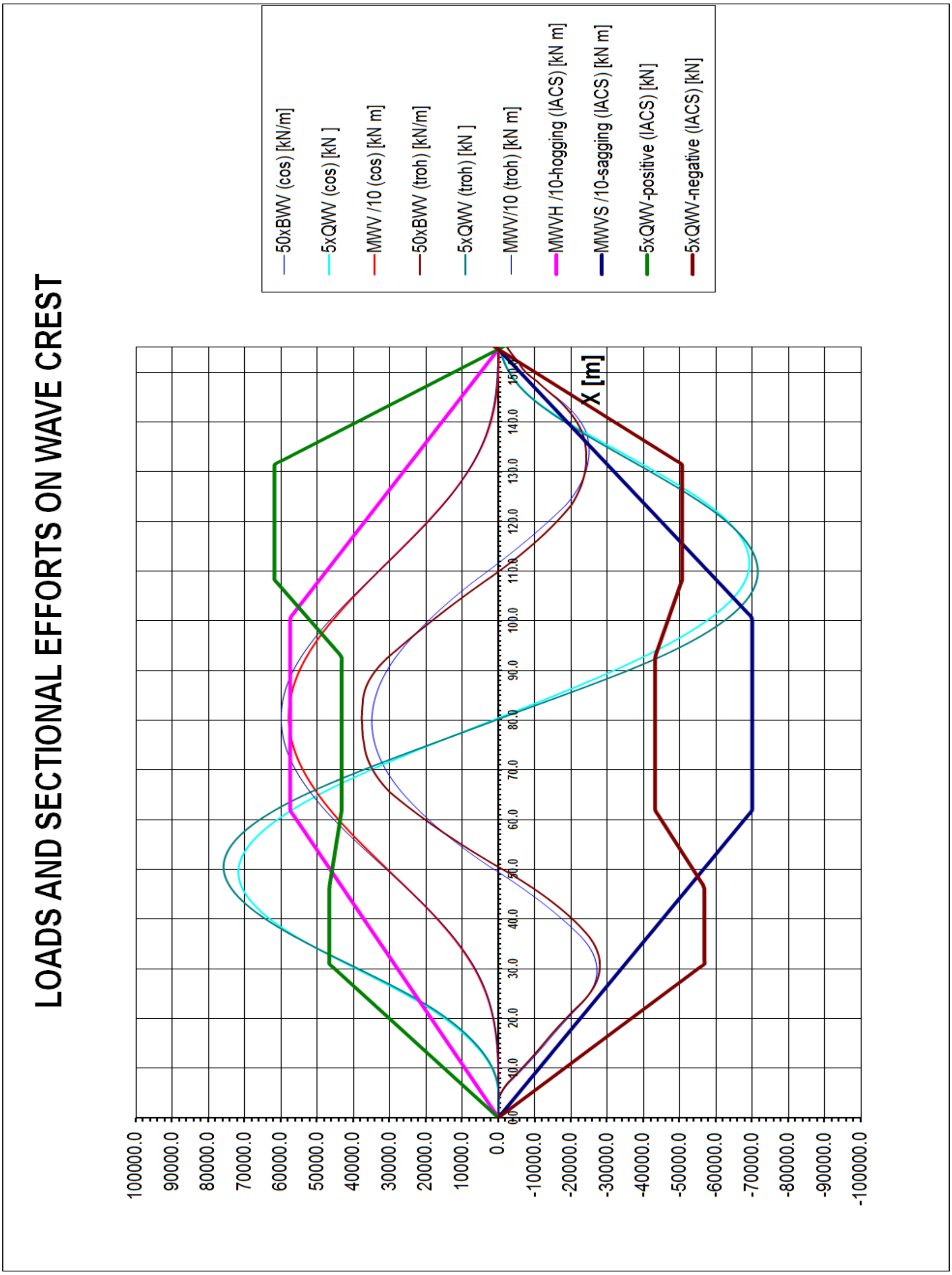


Fig. 3.3.2 – Additional sectional efforts in the hull of the cargo ship of 15000 tdw that was analyzed, for quasi-static layout on wave crest, and of the sectional efforts determined by IACS formulas

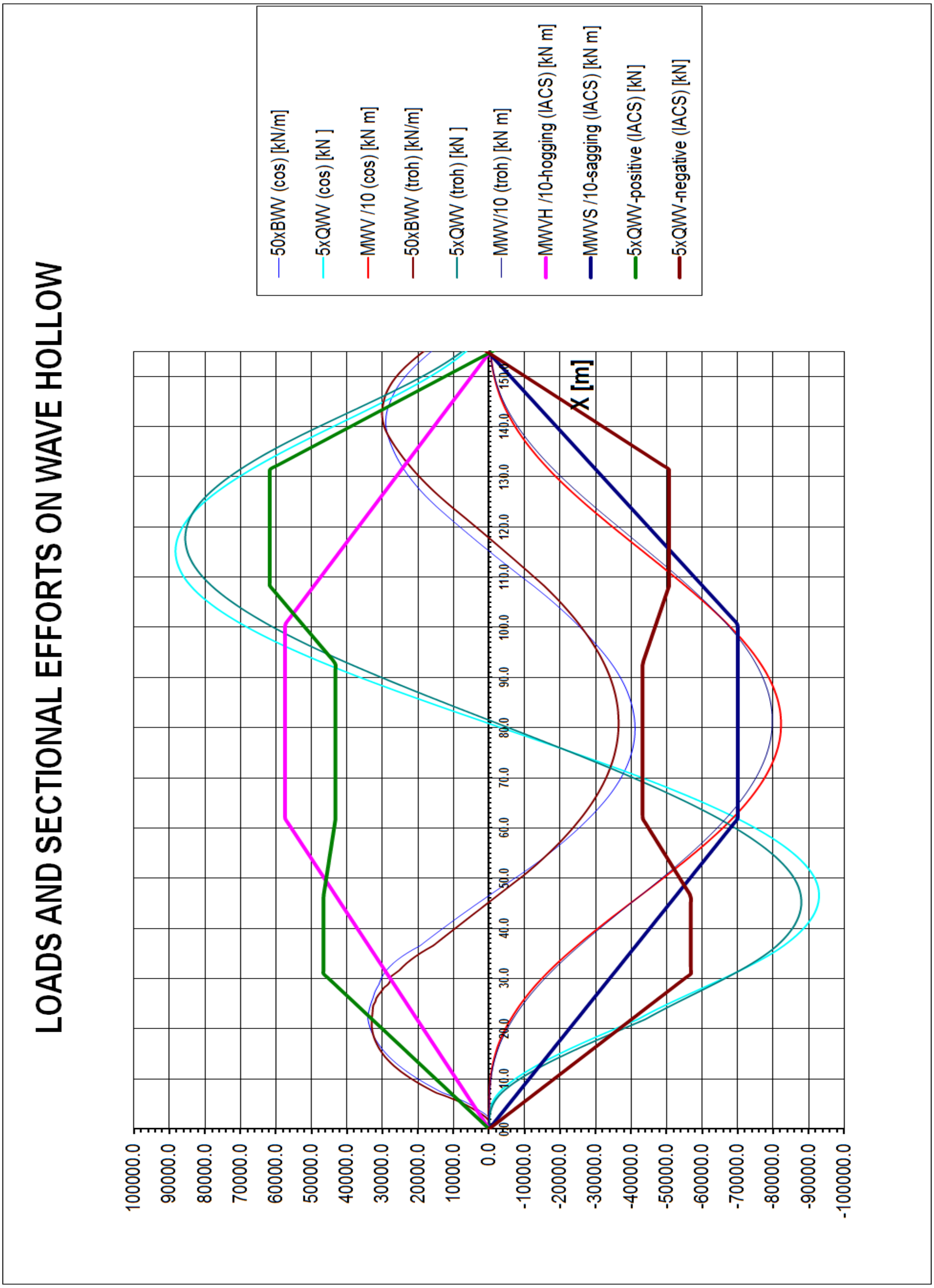


Fig. 3.3.3 – Additional sectional efforts in the hull of the cargo ship of 15000 tdw that was analyzed, for quasi-static layout on wave hollow, and of the sectional efforts determined by IACS formulas

3.3.2 Verification of the IACS method for a 65,000 tdw bulk carrier

The IACS method was also verified for a bulk carrier of 65,000 tdw presented in Figure 3.3.4. The main characteristics of the ship are shown below:

Lmax = 254.10 m
L = 250.00 m
B = 32.20 m
D = 17.00m
T = 12.30m

This one was placed quasi-statically on a wave with the height corrected by the Smith effect, equal to the value C determined by the formula (3.2.3), i.e. equal to 10,396 m and a length equal to the length of the ship (the real wave having a height of 13 m, the period of 12,65 s, the length of 250 m and the speed of 19,8 m/s, occurs with a probability of 0,017%, as indicated by the statistical measurements presented in [75]).

The results of the calculations are shown in Table 3.3.2 and graphically in Figures 3.3.5 and 3.3.6.

In the same table and graph are gived also the additional wave-induced efforts according to the IACS method [74.S11].

Table 3.3.2 – Comparative results of the calculations of the additional sectional efforts induced by waves for the 65000 tdw bulk carrier

Equilibrium parameters, additional efforts induced by waves	**Quasi-static layout on the cosine wave**		**Quasi-static layout on the trochoidal wave**		**IACS Method**		**Differences in [%] between layout on the cosine wave and trochoidal wave**		**Differences in [%] between IACS methode and quasi-static layout on the trochoidal wave**	
	On the crest	On the hollow	On the crest	On the hollow	Hogging bending	Sagging bending	On the crest	On the hollow	Hogging/ On the crest	Sagging/ On the hollow
T_0 *[m]*	10.116	12.734	10.623	13.148	-	-	-4.773	-3.149	-	-
ψ *[rad]*	0.930	-0.922	0.958	-0.904	-	-	-2.923	1.991	-	-
Θ *[rad]*	0	0	0	0	0	0	0.000	0.000	0.000	0.000
$Q_{WV}(+)$ *[kN]*	50134	55983	51541	54467	37737	37737	-2.730	2.783	-36.579	-44.333
$Q_{WV}(-)$ *[kN]*	-48828	-56032	-49851	-54421	-35015	-35015	-2.052	2.960	-42.370	-55.422
M_{WV} *[kNm]*	3468118	-4132476	3547379	-4024886	3209681	-3459289	-2.234	2.673	-10.521	-16.350

It is found that the maximum additional bending moments determined by static laying of the ship on wave are up to 16.35% higher than those determined by IACS, and in the case of shear forces, the differences are much higher, reaching up to 55.5%, which means that the relations indicated by IACS lead to sub-dimensioning of ships in terms of wave-induced sectional efforts.

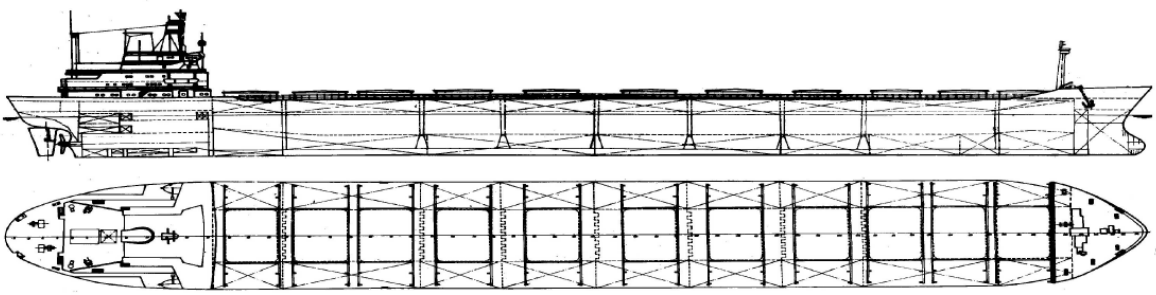


Fig. 3.3.4 – The bulk carrier of 65000 tdw that was analyzed

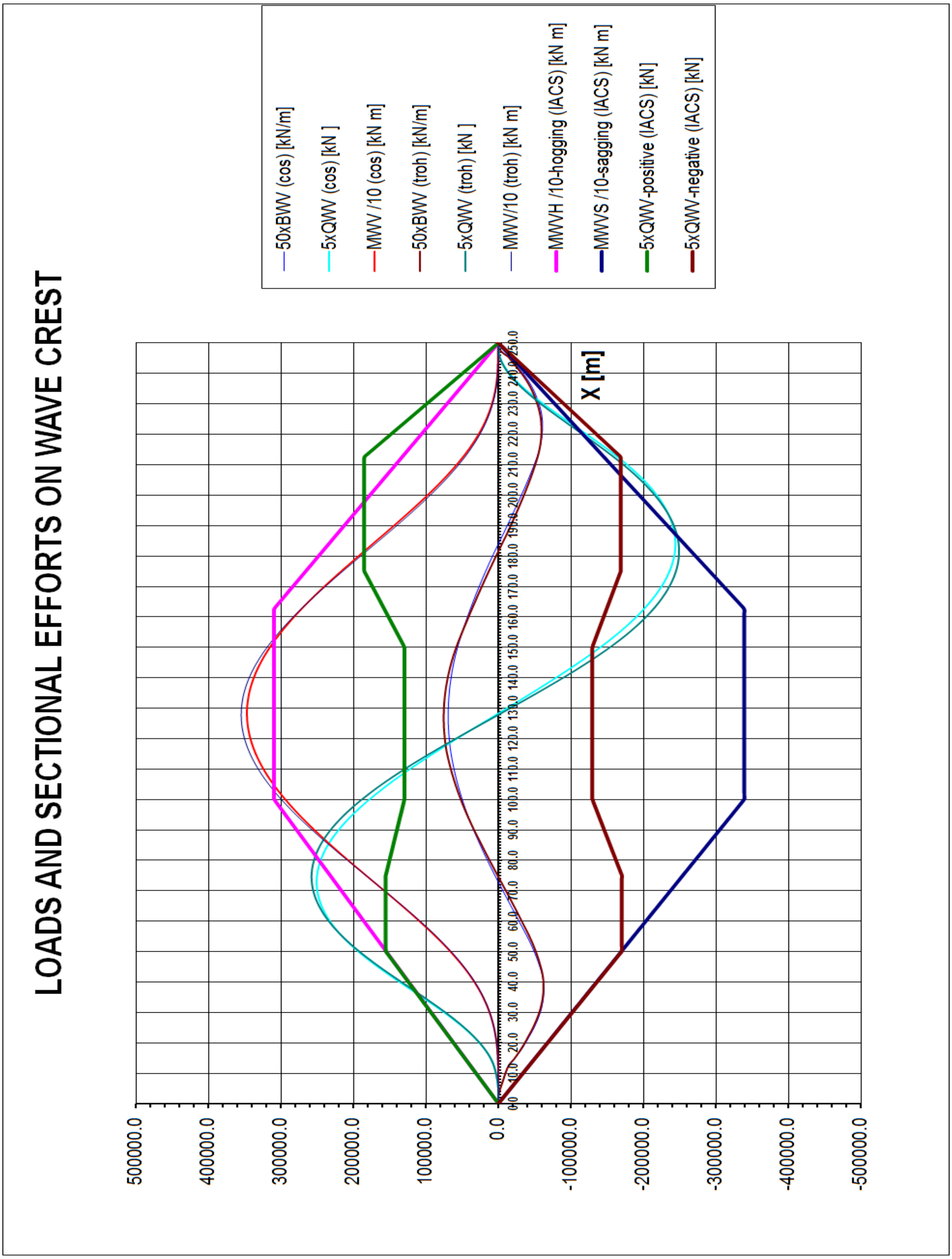


Fig. 3.3.5 – Additional sectional efforts in the hull of the bulk carrier of 65000 tdw that was analyzed, for quasi-static layout on wave crest, and of the sectional efforts determined by IACS formulas

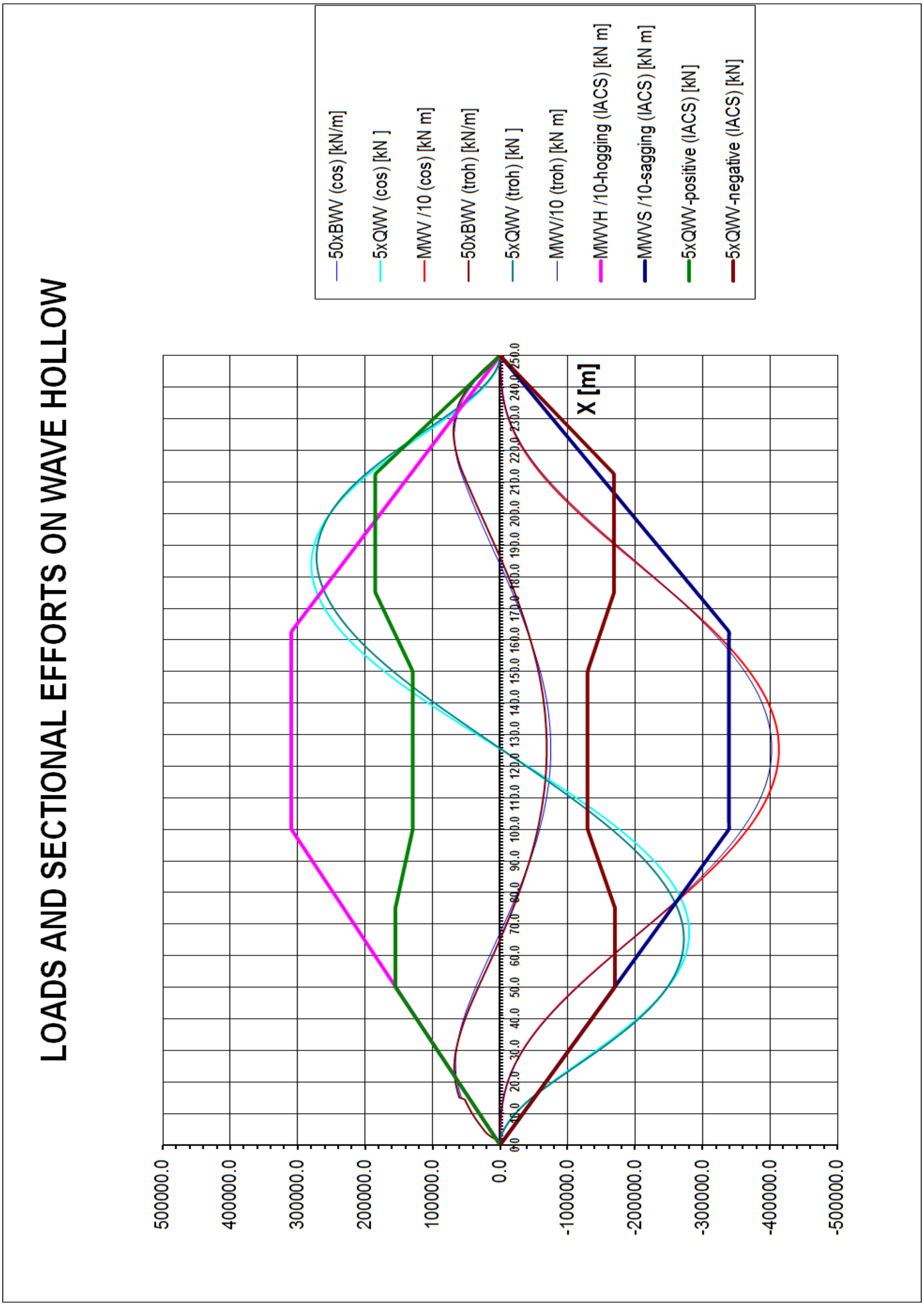


Fig. 3.3.6 – Additional sectional efforts in the hull of the bulk carrier of 65000 tdw that was analyzed, for quasi-static layout on wave hollow, and of the sectional efforts determined by IACS formulas

3.4 Verification of the IACS method for determining of the wave-induced sectional efforts of the seagoing ships hull based on the methods for the dynamic laying of the ship on the wave

The verification of the wave-induced sectional efforts, determined according to IACS, was also performed for the dynamic laying of the ship on the wave, based on the methods presented in 2.3-2.5 and using the RLD-V1, RLD-V1N and RLD-V2 programs.

For this purpose, the comparative calculations presented below were performed, for the two types of ships described in 3.3.

The values of the bending moments determined by the linear method with the RLD-V1 program were corrected due to the non-linearity of the phenomenon, with the formulae indicated in [75], which were determined considering the IACS method [74]:

$$M_h = M_L \cdot \frac{2}{1+R} \tag{3.4.1}$$

$$M_s = M_L \cdot \frac{2 \cdot R}{1+R} \tag{3.4.2}$$

where:

$$R = \frac{C_B + 0.7}{1.73 \cdot C_B} \tag{3.4.3}$$

M_L – the bending moment determined by the linear method with the RLD-V1 program;

M_h – hogging bending moment;

M_s – sagging bending moment;

3.4.1 Verification of the IACS method for 15000 tdw general cargo ship

For the verification of the IACS method, the additional sectional efforts in ship's hull, induced by waves at the dynamic layout, were first determined for the 15,000 tdw general cargo ship presented in Figure 3.3.1.

Based on the significant wave having the height corrected with the Smith effect, equal to the C value determined by the formula (3.2.3), i.e. equal to 8.997 m, and having the length equal to the length of the ship, the parameters of the real wave unaffected by the Smith effect are obtained and used later in dynamic calculation. This wave has the height of 11.000 m, the period of 9.96 s, the length of 155 m and the speed of 15.6 m/s and occurs with a probability of 0.04%, as indicated by the statistical measurements presented in [76].

By dynamically laying the ship on such a wave, by calculations are obtained the results presented graphically in figures 3.4.1 and 3.4.2.

At the same time, the additional wave-induced efforts according to the IACS [74.S11] method, and those additional at the quasi-static layout are given.

It is found that the additional sectional efforts obtained by direct calculations, at the dynamic layout of the ship on the wave, increase with the speed until the number Froude *Fn* exceeds the value of 0.20, after which the tendency is to stabilize and then to decrease. As in quasi-static layout, these efforts are greater than those obtained according to the

IACS method. Thus, the maximum additional bending moments are on average 50% higher than those determined according to IACS at speed above 20 Nd, and in the case of shear forces, the differences reaching up to 80%, which means that the formulae indicated by IACS lead to subdimensioning of the ships in terms of longitudinal strength.

However, up to *Fn* = 0.20, i.e. up to 15 Nd, the moments induced by the dynamic wave are on average 25% higher than those of IACS, and the shear forces by 50%.

Note that up to *Fn* = 0.15, the sectional efforts determined at the quasi-static layout are generally higher than those determined at the dynamic layout.

It is also noted that up to *Fn* = 0.10, dynamically calculated setional efforts are generally below the IACS values.

3.4.2 <u>Verification of the IACS method for 65000 tdw bulk carrier</u>

For the verification of the IACS method, the additional sectional efforts in ship's hull, induced by waves at the dynamic layout, were also determined for the 65000 tdw bulk carrier presented in Fig. 3.3.6.

Based on the significant wave having the height corrected with the Smith effect, equal to the C value determined by the formula (3.2.3), i.e. equal to 10.396 m, and having the length equal to the length of the ship, the parameters of the real wave unaffected by the Smith effect are obtained and used later in dynamic calculation. This wave has the height of 13.000 m, the period of 12.65 s, the length of 250 m and the speed of 19.8 m/s and occurs with a probability of 0.017%, as indicated by the statistical measurements presented in [76].

By dynamically laying the ship on such a wave, by calculations are obtained the results presented graphically in figures 3.4.3 and 3.4.4.

At the same time, the additional wave-induced efforts according to the IACS [74.S11] method, and those additional at the quasi-static layout are given.

It is found that the additional sectional efforts obtained by direct calculations, at the dynamic layout of the ship on the wave, increase with the speed until the number Froude *Fn* exceeds the value of 0.20, after which the tendency is to stabilize and then to decrease. As in quasi-static layout, these efforts are greater than those obtained according to the IACS method. Thus, the maximum additional bending moments are on average 30% higher than those determined according to IACS at speed above 20 Nd, and in the case of shear forces, the differences reaching up to 70%, which means that the formulae indicated by IACS lead to subdimensioning of the ships in terms of longitudinal strength.

Note that up to *Fn* = 0.15, i.e. up to 15 Nd, the sectional efforts determined at the quasi-static layout are generally higher than those determined at the dynamic layout.

It is also noted that up to *Fn* = 0.10, dynamically calculated setional efforts are generally below the IACS values.

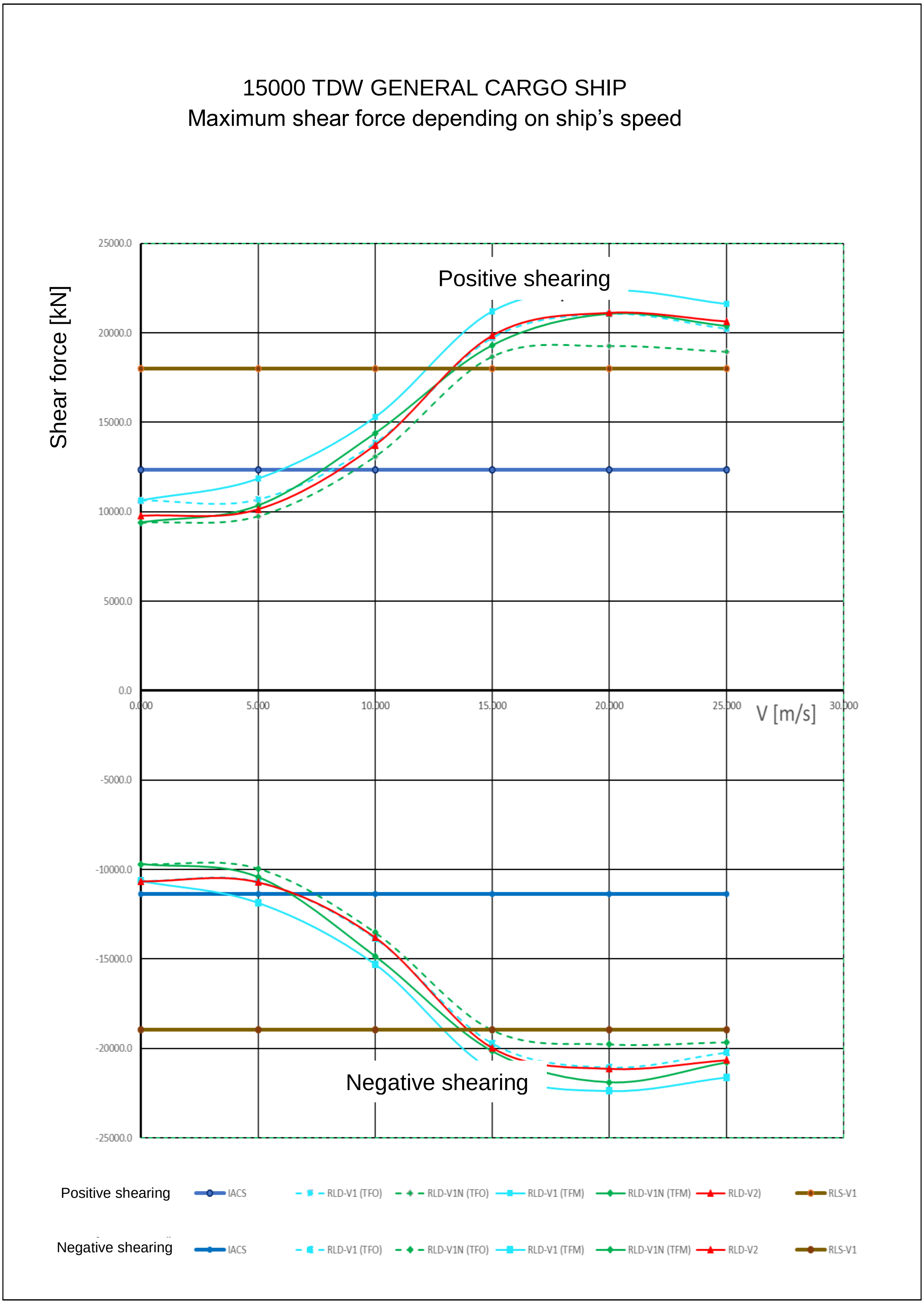


Fig. 3.4.1 – Maximum values of the additional wave-induced shear forces in the hull of 15000 tdw cargo ship, depending on ship speed

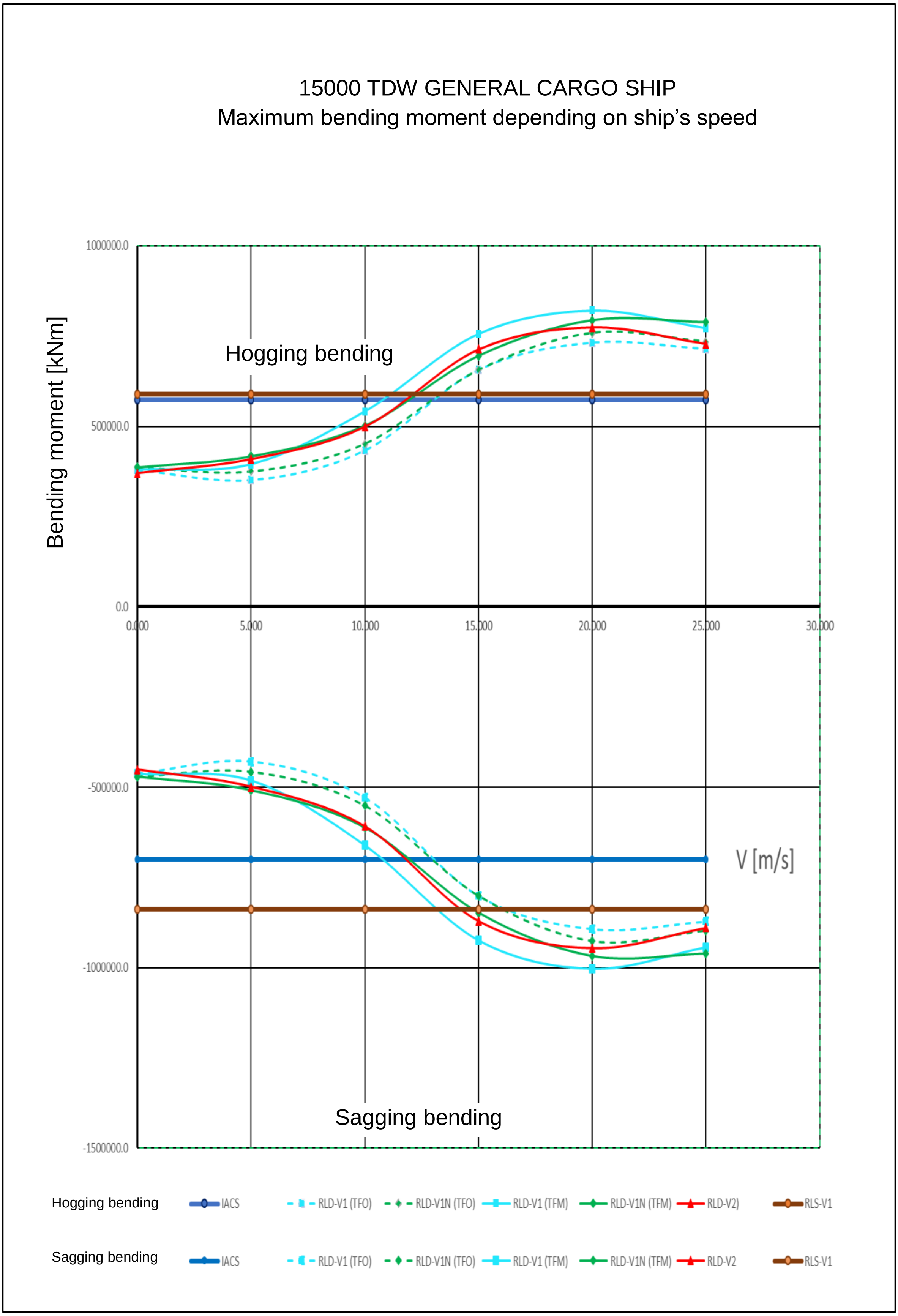


Fig. 3.4.2 – Maximum values of the additional wave-induced bending moments in the hull of 15000 tdw cargo ship, depending on ship speed

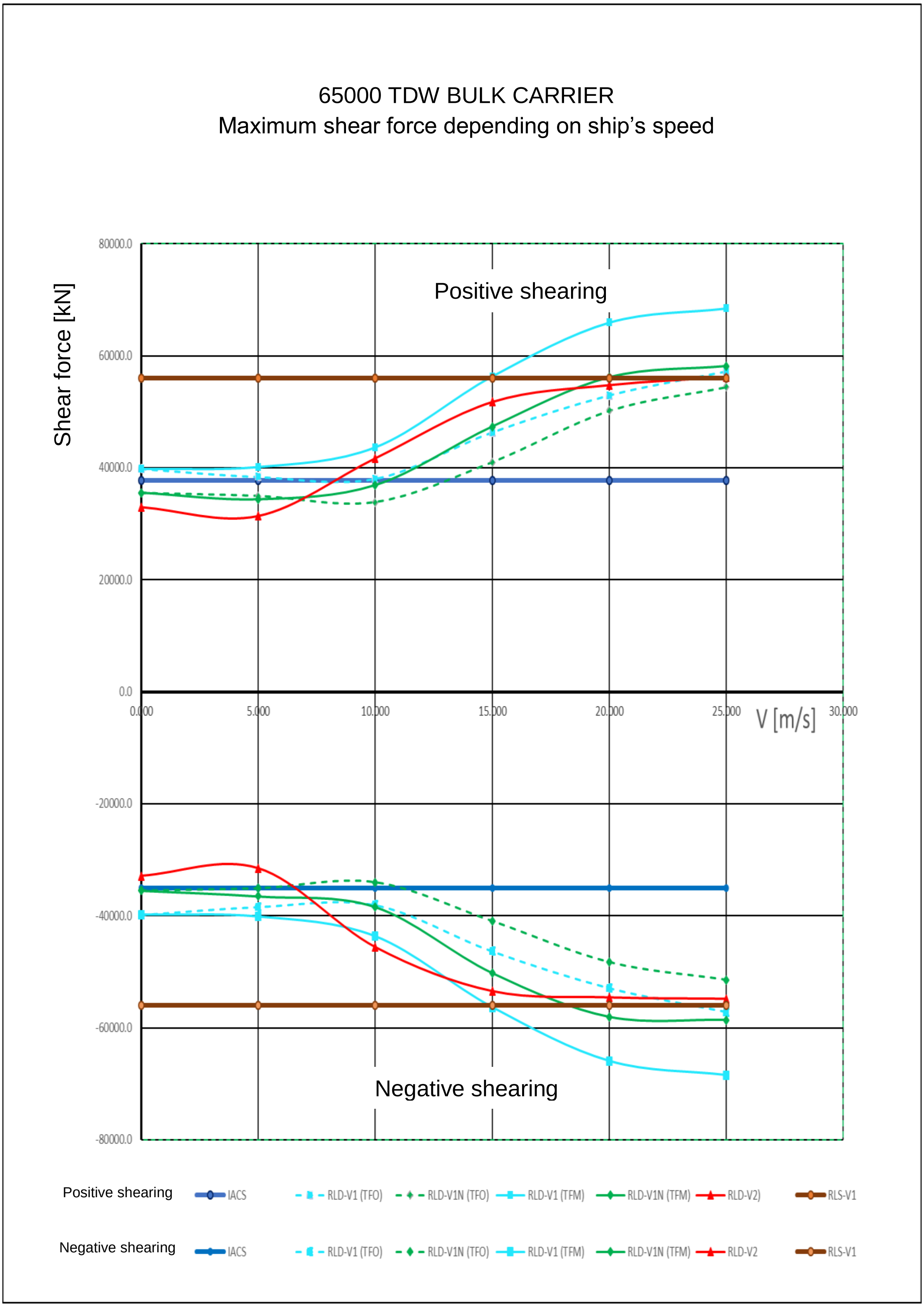


Fig. 3.4.3 – Maximum values of the additional wave-induced shear forces in the hull of 65000 tdw bulk carrier, depending on ship speed

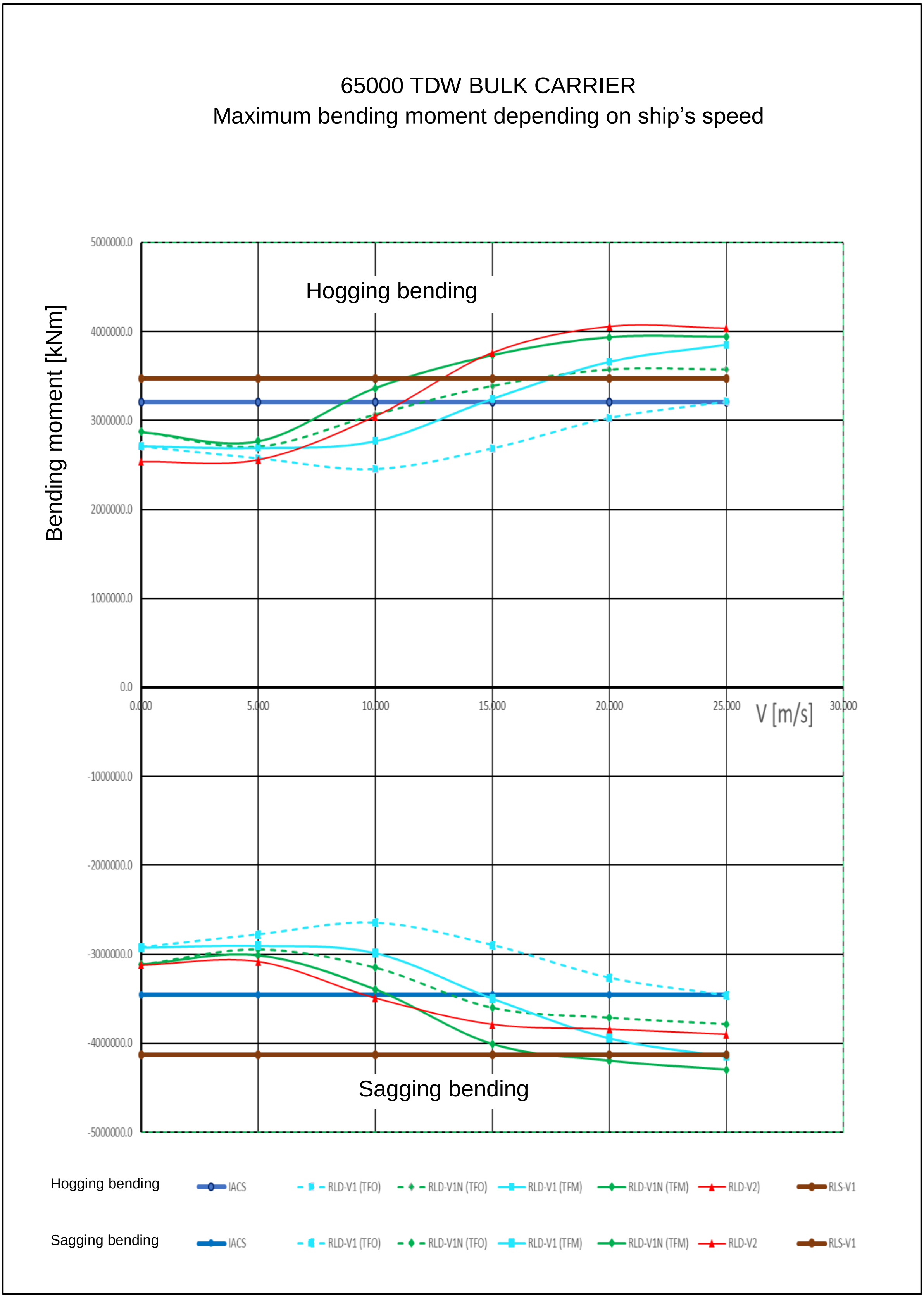


Fig. 3.4.4 – Maximum values of the additional wave-induced bending moments in the hull of 65000 tdw bulk carrier, depending on ship speed

3.5 Comments, conclusions and proposals

From the data presented in 3.4, it follows that the formulas established by IACS for calculating the wave-induced sectional efforts in the seagoing ship's hull are exceeded and lead to sub-dimensioning of their longitudinal structural elements.

This finding is certain because the sectional efforts conforming to IACS were first compared in 3.3 with the values obtained from the quasi-static layout of the ship on the wave which provide high accuracy values. This case of navigation is frequently encountered at navigation with astern waves.

The dynamic layout of the ship on wave has certain approximation limits of efforts, established by testing the calculation methods and programs presented in 2.3, 2.4 and 2.5, on three test models, but the differences between the calculations and the IACS method exceed these limits, so that even in the dynamic layout, the calculated sectional efforts are certainly higher than those according to IACS.

Sub-dimensioning of the ships in terms of wave-induced sectional efforts is confirmed by the data from MSC 75/5/2 report regarding bulk carriers safety analysis in 1978-2000 period, conducted by Japan for the Maritime Safety Committee of IMO [77].

As a result, this organization has imposed, as of July 2006, by SOLAS Regulation XII/6.2, that bulk carriers with a length of more than 150 m have a double side skin [4].

Considering those presented above, in order to improve the construction safety of the ship, the following are proposed:

- the factor k_H in formula (3.2.1) to be increased by approximately 10%, that is, to have the value of 210 instead of 190;
- the factor k_S in formula (3.2.2) to be increased by approximately 20%, that is, to have the value of 130 instead of 110;
- the factor k_Q in formula (3.2.4) to be increased by approximately 50%, that is, to have the value of 50 instead of 30;
- the formulas will be multiplied by the factor k_F which introduces the dependence of *Fn*:

$$k_F = \max(1, \sqrt{0.5 + 6 \cdot F_n - 11 \cdot F_n^2}) \qquad (3.5.1)$$

These proposals modify the IACS formulas for determining wave-induced sectional efforts, as follows:

- for hogging bending moment:

$$M_{WV,H} = \kappa_F \cdot \kappa_H \cdot F_M \cdot C \cdot L^2 \cdot B \cdot C_B \cdot 10^{-3} \text{ [kN m]} \qquad (3.5.2)$$

- for sagging bending moment:

$$M_{WV,S} = \kappa_F \cdot k_S \cdot F_M \cdot C \cdot L^2 \cdot B \cdot (C_B + 0.7) \cdot 10^{-3} \text{ [kN m]} \qquad (3.5.3)$$

- for shear force:

$$Q_{WV} = \kappa_F \cdot k_Q \cdot F_Q \cdot C \cdot L \cdot B \cdot (C_B + 0.7) \cdot 10^{-2} \text{ [kN]} \qquad (3.5.4)$$

where:

$k_H = 210$ – hogging bending coefficient;

$k_S = 130$ – sagging bending coefficient;

$k_Q = 50$ – shear coefficient;

By applying these revised formulas, for the 15000 tdw cargo ship, the new values of IACS wave-induced sectional efforts are shown graphically in Fig. 3.5.1 and 3.5.2 compared to the values determined by direct calculations.

For the 65000 tdw bulk carrier, the results of the same calculations are shown graphically in Fig. 3.5.3 and 3.5.4.

From the analysis of these graphically presented results it is found that the revised IACS formulas sufficiently cover the values determined by direct calculations.

From the performed analysis, it follows that the current IACS formulas for determining the additional wave-induced sectional efforts need to be corrected because these efforts are significantly exceeded in real situations, and in particular the shear forces.

An additional confirmation is also provided by the data in Table 3.5.1 where the sectional efforts determined by direct calculation are compared with those conforming to the current and revised IACS formulas for 2 oil tankers presented in [75]. It is found that the results of direct calculations exceed by up to 30% the values obtained with the current IACS formulas and only up to 10% those obtained with the proposed formulas (3.5.2), (3.5.3) and (3.5.4).

Table 3.5.1 – Comparative results for 2 oil tankers presented in [75] to review revised IACS formulas

Ship's type	**Bending type**	**BENDING MOMENTS [kNm]**		
		current IACS	**revised IACS**	**Direct linear calculation**
65200 tdw oil tanker	hogging	1785670.366	2213087.057	2145000
	sagging	1921743.406	2546698.778	2273000
166300 tdw oil tanker	hogging	5781138.325	6935782.479	7512000
	sagging	6211134.201	7967792.027	7962000

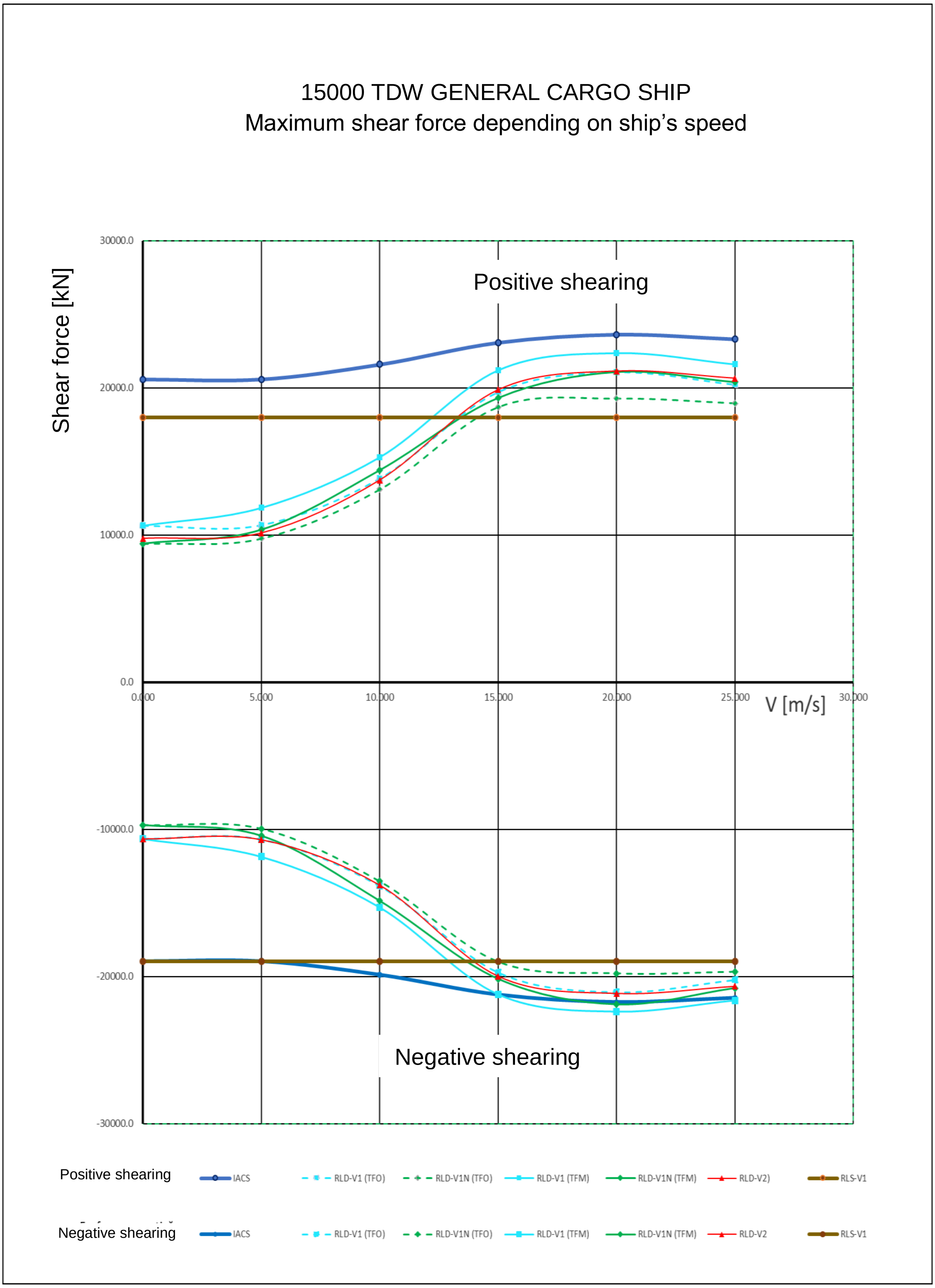


Fig. 3.5.1 – Revised maximum values of the IACS additional wave-induced shear forces in the hull of 15000 tdw cargo ship, compared to the values determined by direct calculations

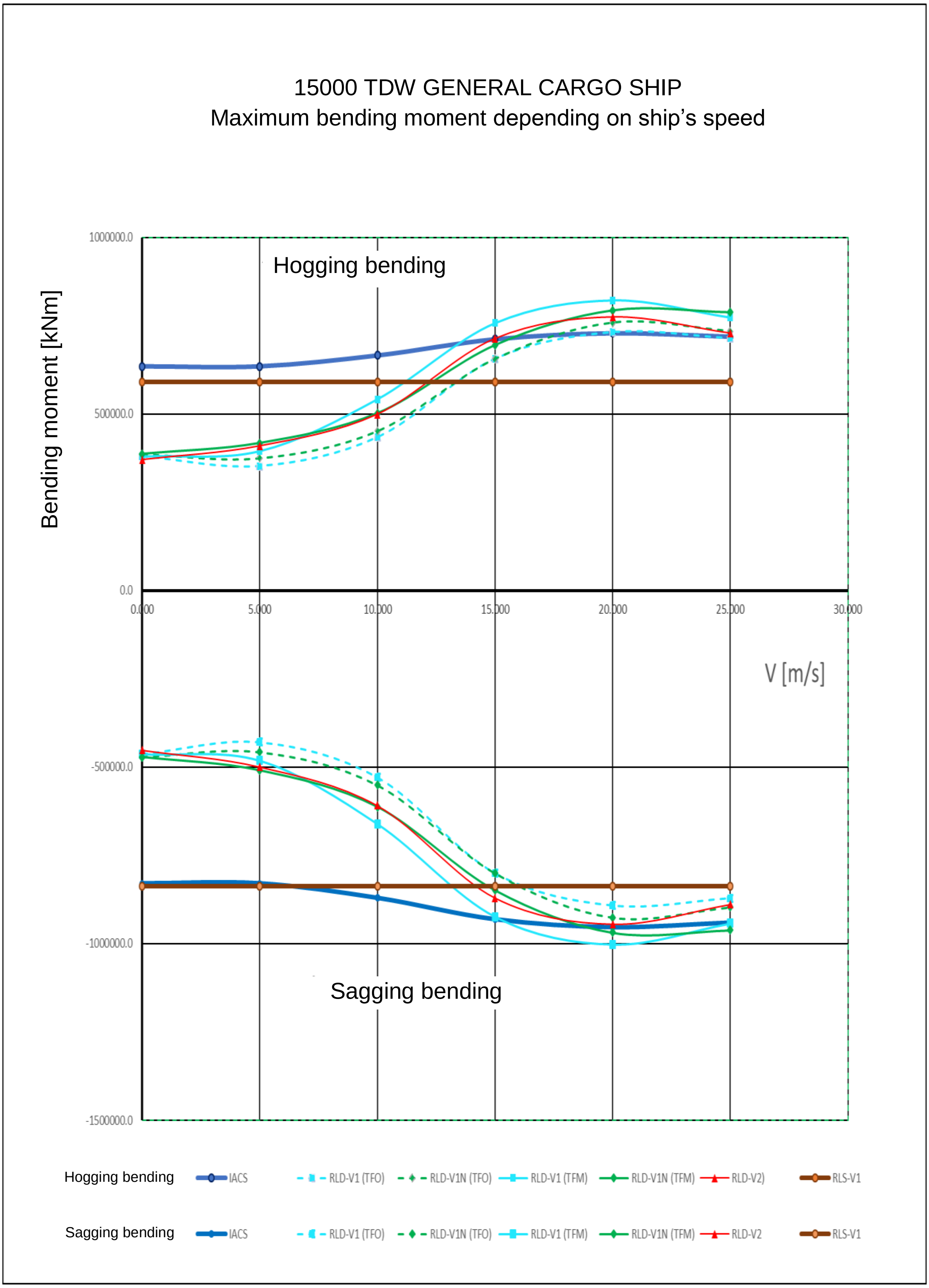


Fig. 3.5.2 – Revised maximum values of the IACS additional wave-induced bending moments in the hull of 15000 tdw cargo ship, compared to the values determined by direct calculations

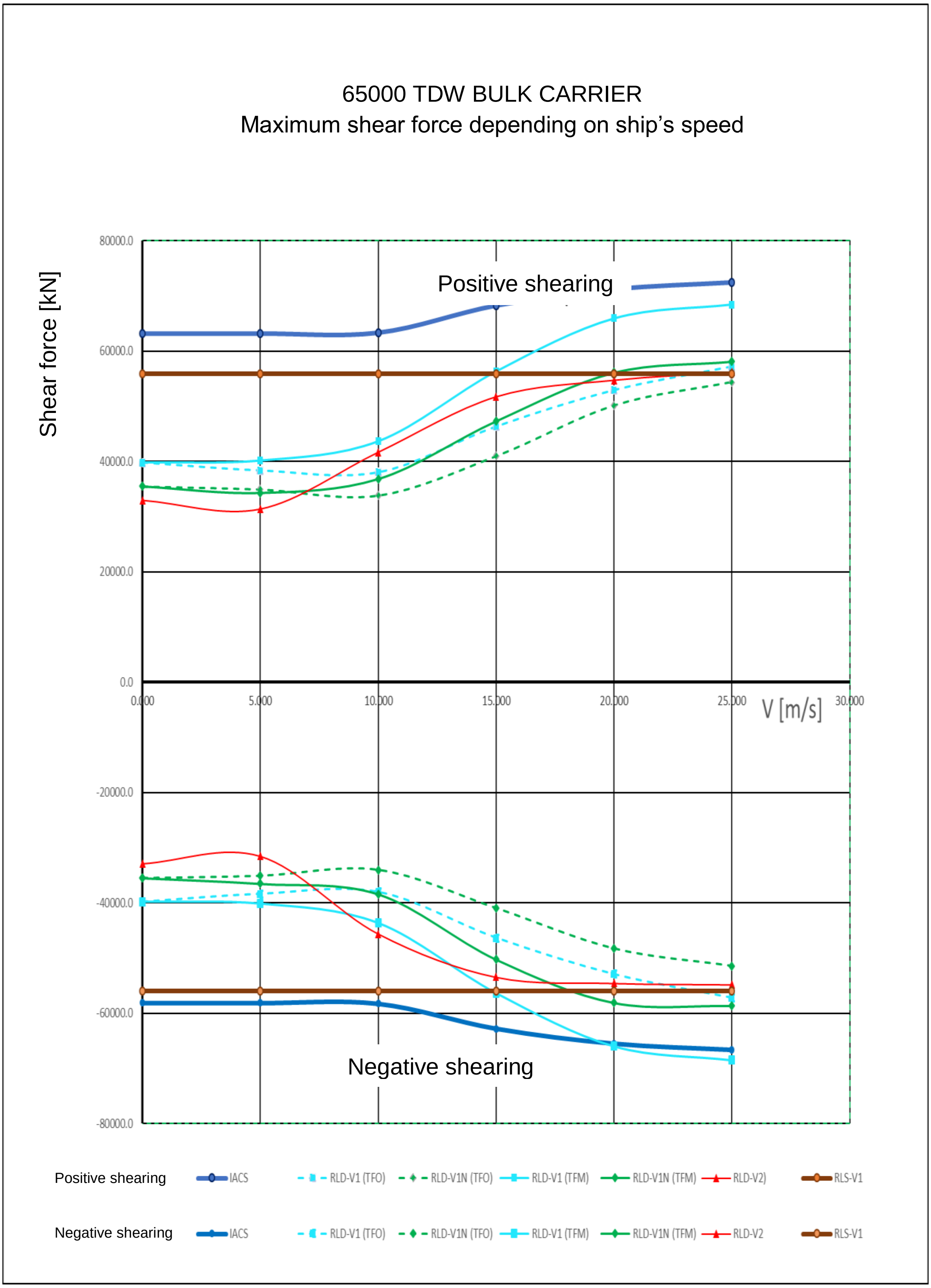


Fig. 3.5.3 – Revised maximum values of the IACS additional wave-induced shear forces in the hull of 65000 tdw cargo ship, compared to the values determined by direct calculations

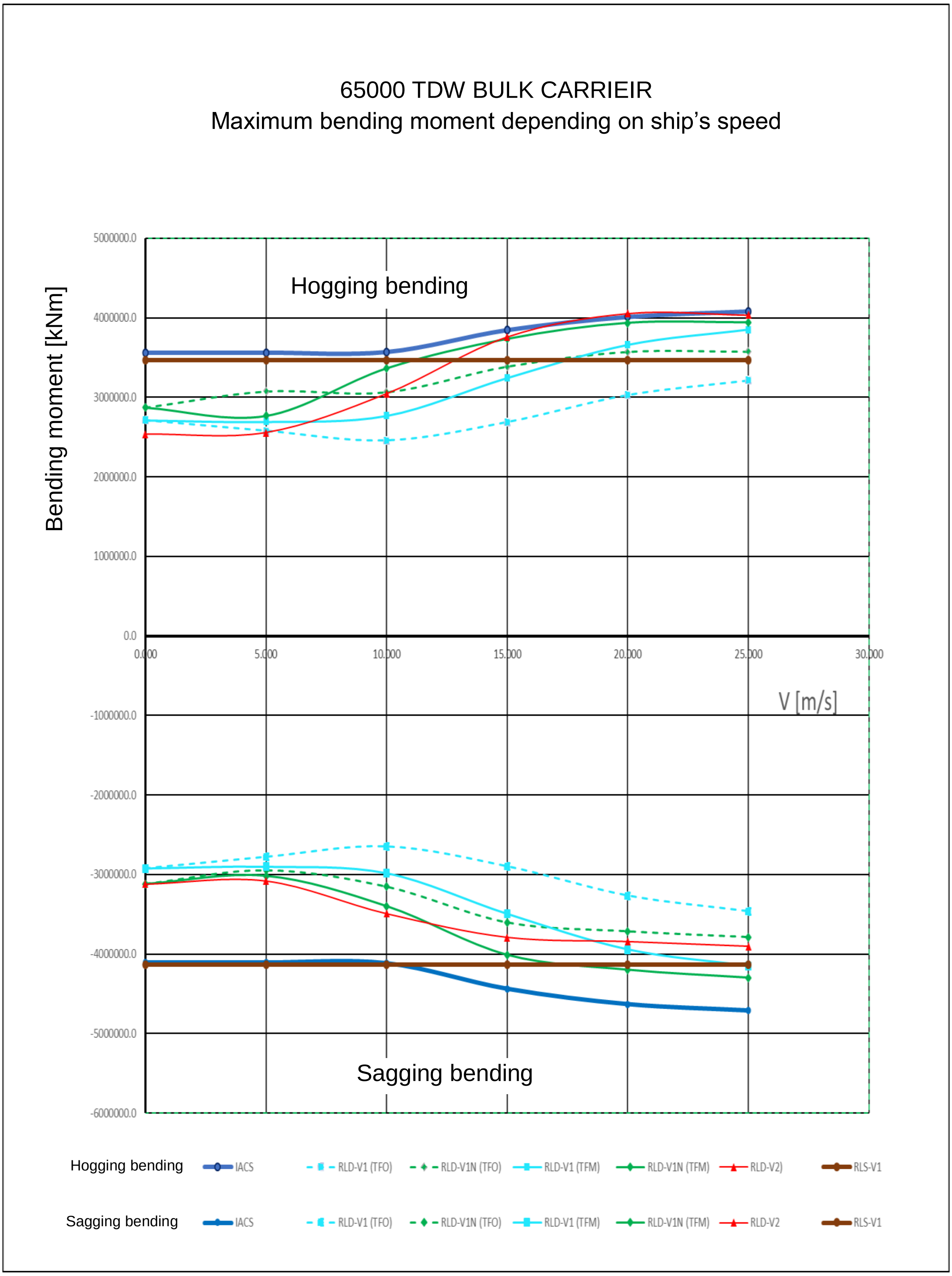


Fig. 3.5.4 – Revised maximum values of the IACS additional wave-induced bending moments in the hull of 65000 tdw cargo ship, compared to the values determined by direct calculations

4 PROBABILISTIC ASSESSMENT OF LONGITUDINAL RESIDUAL STRENGTH OF THE SEAGOING DAMAGED SHIPS

4.1 Presentation of currently applicable criteria

According to the IACS Common Structural Rules [5], it is considered that the safety of the damaged ship is assured if the following deterministic criteria are met for both hogging and sagging bending:

$$\left|\gamma_{SD} \cdot M_{SW-D} + \gamma_{WD} \cdot M_{WV}\right| \le \left|\frac{M_{UD}}{\gamma_{RD} \cdot C_{NA}}\right| \tag{4.1.1}$$

where

M_{SW-D} – permissible still water bending moment for navigation with ship in the damaged condition, in the current section, [kN m];

M_{WV} – wave-induced bending moment for navigation with ship in the intact condition, [kN m];

M_{UD} – Vertical hull girder ultimate bending capacity in the damaged condition [kN m];

γ_{SD} – safety factor for the still water bending moment in the damaged condition: $\gamma_{SD} = 1.1$

γ_{WD} – safety factor for the vertical wave bending moment in the damaged condition: $\gamma_{WD} = 0.67$

γ_{RD} – safety factor for the vertical hull girder ultimate bending capacity in the damaged condition: $\gamma_{RD} = 1.00$

C_{NA} – neutral axis coefficient: $C_{NA} = 1.00$ for grounding;
$C_{NA} = 1.10$ for collision.

Because these rules do not address the shear strength in damaged condition of the hull, it is proposed to check this strength after a relation similar to relation (4.1.1), i.e.:

$$\left|\gamma_{SD} \cdot Q_{SWD} + \gamma_{WD} \cdot Q_{WV}\right| \le \left|\frac{Q_{UD}}{\gamma_{RD} \cdot C_{NA}}\right| \tag{4.1.2}$$

where :

Q_{SW-D} – permissible still water shear force for navigation with ship in the damaged condition, in the current section, [kN];

Q_{WV} – wave-induced shear force for navigation with ship in the intact condition, [kN];

Q_{UD} – vertical hull girder ultimate shear force in the damaged condition [kN];

The extent of the damage is considered according to Table 4.1.1 and Fig.4.1.1 for collisions and according to Table 4.1.2 and Fig.4.1.2 for failures.

Table 4.1.1 – The extent of damage caused by collision

Extent of damage caused by collision [m]	Position on side	
	Simple side	Double side
Height, *h*	*0.75 D*	*0.60 D*
Width, *d*	*B/16*	*B/16*
Length, *l*	*0.1 L*	*0.1 L*

Tabel 4.1.2 – The extent of damage caused by grounding

Extent of damage caused by grounding [m]	Position on side	
	Bulk carriers	Oil tankers
Height, *h*	*Min(B/20;2)*	*Min(B/15;2)*
Width, *d*	*0.60 B*	*0.60 B*
Length, *l*	*0.3 L*	*0.3 L*
Note: the ships with simple bottom will have length of the damage: *l = 0.5 L*		

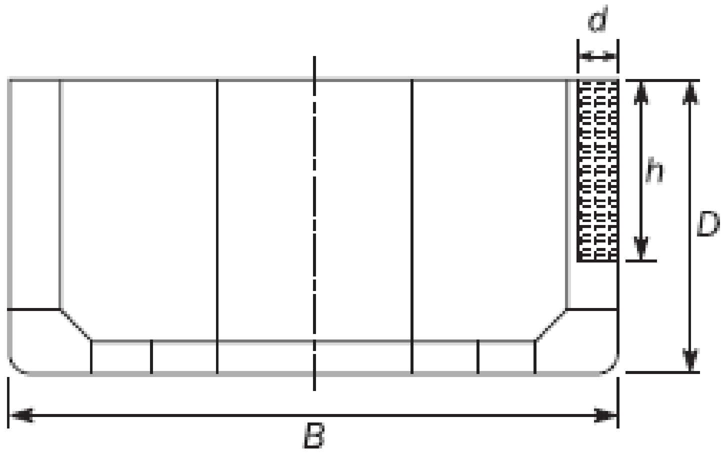


Fig. 4.1.1 – The extent of damage caused by collision [5]

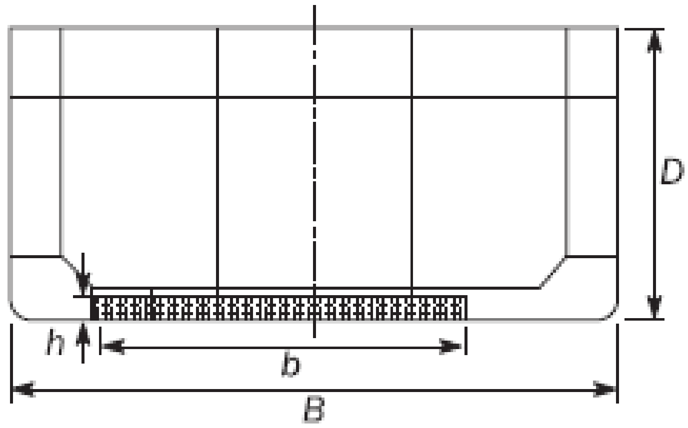


Fig. 4.1.2 – The extent of damage caused by grounding [5]

4.2 Probabilistic assessment of the longitudinal residual strength of the seagoing damaged ships

To solve such a problem, it is proposed to apply a probabilistic concept to treat the construction safety of the ship after damage in terms of longitudinal residual strength, which is based on the ability of survival after damage, as a measure of ship safety assesment in damaged conditions hereinafter referred to as the effective longitudinal residual strength index R_L.

This probabilistic concept on construction safety of the ship after damage concerning longitudinal residual strength is similar to the probabilistic concept for dealing with its post-damage stability, covered by the 1974 SOLAS Convention [4] in Part B-1, Cap. II-1, to determine the probability of survival in terms of stability under damage conditions, characterized by the effective subdivision index.

By the probability theory it can be demonstrated that the effective longitudinal residual strength index R_L of the ship can be calculated for each loading case as the sum of the probability of occurrence of the damage in each compartment and each group of two, three, etc., adjacent compartments multiplied by, respectively, the ship's probability of survival after such damages:

$$R_L = \sum p_i r_i \tag{4.2.1}$$

where:

- i is the index of each compartment or group of compartments considered,
- p_i indicates the probability that only the considered compartment or group of compartments will be damaged;
- r_i indicates probability of survival after damage to the considered compartment or compartment group;

The p_i probability of occurrence of a damage in a certain area of the hull and its dimensions [81], (see fig. 4.2.1) can be determined based on the statistical records on damages from the exploitation of the ships. For the shell, formulas for this probability are indicated in SOLAS Convention [4], in Chapter II-1.

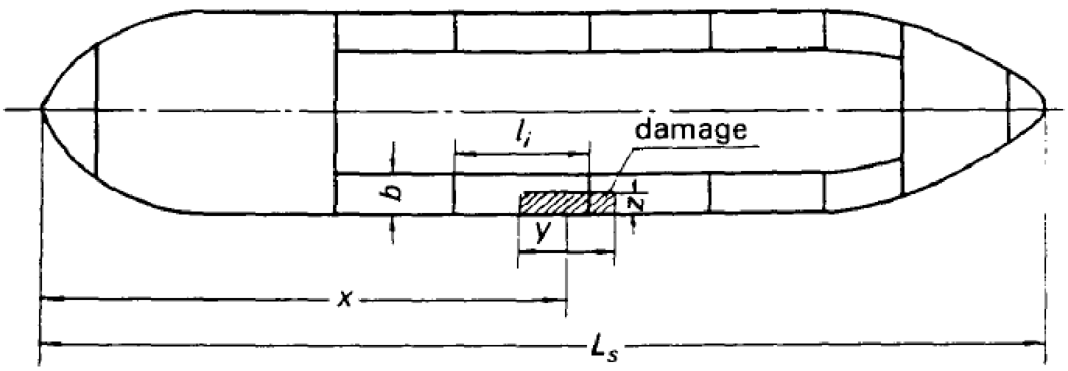


Fig. 4.2.1 – Damage dimensions and location [81]

Probability of survival after collision r_i is proposed to be determined for each considered case of hypothetical damage, in any initial loading situation, with the relation:

$$r_i = \left[Max\left(0;1-\frac{\left|\gamma_{SD}\cdot M_{SW-D}+\gamma_{WD}\cdot M_{WV}\right|}{\left|\dfrac{M_{UD}}{\gamma_{RD}\cdot C_{NA}}\right|}\right)\cdot Max\left(0;1-\frac{\left|\gamma_{SD}\cdot Q_{SW-D}+\gamma_{WD}\cdot Q_{WV}\right|}{\left|\dfrac{Q_{UD}}{\gamma_{RD}\cdot C_{NA}}\right|}\right)\right]^{\frac{1}{4}} \quad (4.2.2)$$

The terms of formula (4.2.2) were defined in 4.1.

To determine the ultimate sectional efforts M_{UD} and Q_{UD} of the damaged cross-section of the hull, it is necessary to know the size of the damage. This must be consistent with the configuration and structure of the damaged compartment and must be of such size as to produce the greatest reduction in the hull strength. For the damage of a side compartment, at least the dimensions of the damages defined in Figure 4.1.1 may be considered.

However, the dimensions of the damage in a side compartment shall not exceed the values covered by SOLAS 1974 [5] and MARPOL [16] in Table 4.2.1, which occur with a probability of approx. 50% at a confidence level of 95%. If it is intended to consider damage with a 97.5% probabilistic coverage level and a confidence level of 95% then the maximum longitudinal extent of damage will be $0.225L_S$ and the transverse extent will be 0.5B

Table 4.2.1 – The extent of damage by collision

Longitudinal extent	1/3 $L^{2/3}$ or 14,5 m, whichever is less
Transverse extent	B/5 or 11.5 m, whichever is less
Vertical extenst	From the baseline, upward, unlimited.

The probabilistic criterion for the construction safety of a seagoing ship, to ensure longitudinal residual strength in damage situations, is proposed to be represented (by similarity with the damage stability probabilistic criterion required by SOLAS [4], in Chapter II-1, Part B-1) by condition that the sum of the partial effective strength indices, R_{Ls}, R_{Lp} and R_{L1}, for 3 representative draught, is not less than the required longitudinal residual strength index R_{Lo} and the additional condition that the partial indices R_{Ls}, R_{Lp} and R_{L1} do not be less than $0.9R_{Lo}$ for passenger ships and $0.5R_{Lo}$ for cargo ships, i.e. that the following relations be fulfilled:

$$R_L \geq R_{Lo} \quad (4.2.3)$$

$$\begin{cases} R_{Ls} \geq 0.9\cdot R_{Lo} \\ R_{Lp} \geq 0.9\cdot R_{Lo} \\ R_{Ll} \geq 0.9\cdot R_{Lo} \end{cases} \quad \text{for passenger ships} \quad (4.2.4)$$

$$\begin{cases} R_{Ls} \geq 0.5\cdot R_{Lo} \\ R_{Lp} \geq 0.5\cdot R_{Lo} \\ R_{Ll} \geq 0.5\cdot R_{Lo} \end{cases} \quad \text{for cargo ships} \quad (4.2.5)$$

where:

$$R_L = 0.4\cdot R_{Ls} + 0.4\cdot R_{Lp} + 0.2\cdot R_{Ll} \quad (4.2.6)$$

R_{Ls} – the effective longitudinal residual strength index R_L at *the deepest subdivision draught* d_s considered to be the draught which corresponds to the summer load line of the ship;

R_{Lp} – the effective longitudinal residual strength index R_L at *the* partial subdivision draught, considered to be the light service draught plus 60% of the difference between the light service draught and the deepest subdivision draught.

R_{Ll} – the effective longitudinal residual strength index R_L at *the light service draught* considered to be the service draught corresponding to the lightest anticipated loading and associated tankage, including, however, such ballast as may be necessary for stability and/or immersion. Passenger ships should include the full complement of passengers and crew on board.

The required longitudinal residual strength index R_{Lo}, can be determined according to the IMO Revised guidelines for formal safety assessment for use in the rule-making process. [83].

If the same level of probabilistic safety is required for longitudinal residual strength after damage and for damage stability, then the required residual longitudinal strength index R_{Lo} can be determined with the same formulas of the SOLAS Convention, Chapter II-1, Part B-1, Rule 6.

Similarly, it can be probablilistic verified the longitudinal residual strength of the ship for the bottom damage as a result of the ship's grounding. For the damage of a bottom compartment, the dimensions defined in fig. 4.1.2 can be considered.

The maximum design dimensions for bottom damage are considered to be those ruled by SOLAS 1974 and MARPOL according to the table 3.3.4 below:

Table 4.2.2 – The extent of damage by grounding

	For 0.3 L from the forward perpendicular of the ship	Any other part of the ship
Longitudinal extent	1/3 $L^{2/3}$ or 14,5 m, whichever is less	1/3 $L^{2/3}$ or 14,5 m, whichever is less
Transverse extent	B/6 or 10 m, whichever is less	B/6 or 5 m, whichever is less
Vertical extent, measured from the keel line	B/20 or 2 m, whichever is less	B/20 or 2 m, whichever is less

4.3 Comments and conclusions

The probabilistic assessment method of the longitudinal residual strength of the damaged ships hull proposed in 4.2 is a modern, elegant and synthetic way of assessing the construction safety of ships, in line with the mode of probabilistic analysis of the stability of damaged ships regulated by the 1974 SOLAS Convention [4] and which has proven its effectiveness by its application.

By taking into account of a large number of damage cases whose influence is found in the longitudinal residual strength index R_L, depending on the probability of occurence and the degree of affecting the longitudinal strength, a better assessment of the construction safety of the damaged ship is achieved.

5 PROBABILISTIC ASSESSMENT OF OVERALL SURVIVAL OF THE SEAGOING DAMAGED SHIPS

5.1 Overview

The need for a probabilistic assessment of the overall survival of seagoing vessels has arisen as a result of the fact that their longitudinal residual strength and stability under damaged conditions must be ensured simultaneously for their safe operation. When determining the method of assessment, we considered the ones presented in the chapter. 4 and the tools presented in 2 were used.

5.2 Description of the method for the probabilistic assessment of the overall survival of ships

For the overall probabilistic assessment of the safety of a damaged ship, it is proposed to apply a probabilistic concept, which is based on the overall survival ability of the ship after damage, as a measure of ship safety assesment concerning residual longitudinal strength and stability, hereinafter referred to as the effective survival index S_G.

By the probability theory it can be demonstrated that the effective survival index S_G of the ship, corresponding to a loading case, can be calculated as the sum of the probability of occurrence of the damage in each compartment and each group of two, three, etc., adjacent compartments multiplied by the ship's probabilities of survival after such damages:

$$S_G = \sum p_i r_i s_i \tag{5.2.1}$$

where:

i represents index of each compartment or group of compartments under consideration,

p_i indicate the probability that only the compartment or group of compartments under consideration may be damaged after a collision or grounding. For collisions, p_i is determined in accordance with SOLAS Convention [4], Ch.II-1, Part B-1,

r_i indicate the probability of survival in terms of longitudinal residual strength after damaging of the compartment or group of compartments under consideration and is calculated with the following formula:

$$r_i = \left[Max\left(0;1-\frac{\left|\gamma_{SD}\cdot M_{SW-D}+\gamma_{WD}\cdot M_{WV}\right|}{\left|\dfrac{M_{UD}}{\gamma_{RD}\cdot C_{NA}}\right|}\right)\cdot Max\left(0;1-\frac{\left|\gamma_{SD}\cdot Q_{SW-D}+\gamma_{WD}\cdot Q_{WV}\right|}{\left|\dfrac{Q_{UD}}{\gamma_{RD}\cdot C_{NA}}\right|}\right)\right]^{\frac{1}{4}} \tag{5.2.2}$$

The terms of formula (5.2.2) were defined in 4.1.

s_i indicate the probability of survival in terms of stability, after damaging of the compartment or group of compartments under consideration, and is determined in accordance with SOLAS Convention [4], Ch.II-1, Part B-1;

The overall probabilistic criterion for the construction safety of a seagoing ship, to simultaneous ensure both the overall longitudinal residual strength and the stability in damage situations, is proposed to be represented (by similarity with the damage stability probabilistic criterion required by SOLAS [4], in Chapter II-1, Part B-1) by condition that the sum of the partial effective survival indices, S_{Gs}, S_{Gp} and S_{G1}, for 3 representative draught, is not less than the required overall survival index S_{Go} and the additional condition that the partial indices S_{Gs}, S_{Gp} and S_{G1} do not be less than 0.9S_{Go} for passenger ships and 0.5S_{Go} for cargo ships, i.e. that the following relations be fulfilled:

$$S_G \geq S_{Go} \tag{5.2.3}$$

$$\begin{cases} S_{Gs} \geq 0.9 \cdot S_{Go} \\ S_{Gp} \geq 0.9 \cdot S_{Go} \\ S_{Gl} \geq 0.9 \cdot S_{Go} \end{cases} \quad \text{for passenger ships} \tag{5.2.4}$$

$$\begin{cases} S_{Gs} \geq 0.5 \cdot S_{Go} \\ S_{Gp} \geq 0.5 \cdot S_{Go} \\ S_{Gl} \geq 0.5 \cdot S_{Go} \end{cases} \quad \text{for cargo ships} \tag{5.2.5}$$

where:

$$S_G = 0.4 \cdot S_{Gs} + 0.4 \cdot S_{Gp} + 0.2 \cdot S_{Gl} \tag{5.2.6}$$

S_{Gs} – the effective overal survival index S_G at *the deepest subdivision draught* d_s considered to be the draught which corresponds to the summer load line of the ship;

S_{Gp} – the effective overal survival index S_G at *the* partial subdivision draught, considered to be the light service draught plus 60% of the difference between the light service draught and the deepest subdivision draught.

S_{Gl} – the effective overal survival index S_G at *the light service draught* considered to be the service draught corresponding to the lightest anticipated loading and associated tankage, including, however, such ballast as may be necessary for stability and/or immersion. Passenger ships should include the full complement of passengers and crew on board.

The required overal survival index S_{Go} can be determined according to the IMO Revised guidelines for formal safety assessment for use in the rule-making process. [83].

If the same level of probabilistic safety is required for overall survival after damage as for only damage stability, then the required overal survival index S_{Go} can be determined with the same formulas of the SOLAS Convention, Chapter II-1, Part B-1, Rule 6.

5.3 Comments and conclusions

The assessment based on overall probabilistic criterion of survival of the damaged ships, proposed in 5.2 is a modern, elegant and synthetic way of assessing the construction safety of ships, in line with the mode of probabilistic analysis of the stability of damaged ships regulated by the 1974 SOLAS Convention [4] and which has proven its effectiveness by its application.

By taking into account of a large number of damage cases whose influence is found in the effective survival index S_G, depending on the probability of occurence and the degree of affecting the longitudinal residual strength, a better assessment of the construction safety of the damaged ship is achieved.

6 GENERAL CONCLUSIONS, ORIGINAL CONTRIBUTIONS AND PERSPECTIVES

6.1 General conclusions

The main scop of the thesis was, as a result of documentation and research, to make proposals to improve the requirements on construction safety of the ships set out in international and national regulations and to develop methods and calculation tools that allow assessment of the construction safety of the ships.

In this respect, the following were achieved:

1. analyzing the current state of regulations concerning construction safety of ships and how they are being implemented from the design phase, continuing during construction, until the operational phase;
2. proposals to improve the construction safety of the seagoing ship by :
   - increasing the longitudinal strength of their hull as a result of the revision of the current method in international regulations on calculating the wave-induced sectional efforts. This review is necessary because the analysis of the results of the calculations made by the current IACS formulas for determining these efforts and those determined by direct calculations revealed that the longitudinal structure of the hull of the vessels is under-dimensioned, in particular at shear, this being confirmed by the significant loss of simple hull bulk carriers. For these reasons, the IMO have imposed that vessels of this type, which exceed 150 meters in length, be with double hull;
   - the probabilistic evaluation of their longitudinal residual strength in damage situations. Such an assessment would be a modern, elegant and synthetic way of assessing the safety of their construction, in line with the mode of probabilistic analysis of the stability of damaged ships, regulated by the 1974 SOLAS Convention [4], and which has proven its effectiveness by its application. Taking into account a large number of damage cases, whose influence is found in the effective residual longitudinal strength index R_L, depending on the probability of occurrence and the degree of affecting the longitudinal strength, allows for a better assessment of the ship's safety damaged;
   - the probabilistic evaluation of their overall survival in damage situations. Such an assessment would be a modern, elegant and synthetic way of assessing the safety of their construction, in line with the mode of probabilistic analysis of the stability of damaged ships, regulated by the 1974 SOLAS Convention [4], and which has proven its effectiveness by its application. Taking into account a large number of damage cases, whose influence is found in the effective overall survival index S_G, depending on the probability of occurrence and the degree of affecting the longitudinal residual strength and stability, allows for a better assessment of the ship's safety damaged;
3. adopting in a specific way, in order to automate calculations, of a method of determining the ship's oscillation parameters and the sectional efforts in the ship's hull for layout on still water and for the quasi-static layout on the cosine and trohoidal wave, as well as the elastic line of the hull, on the basis of which the RLS-V1 program was developed. The validation of the method and program was done by direct calculations. These tools proved to be particularly effective in the research undertaken to establish proposals to improve the construction safety of the ship in terms of longitudinal strength;

4. adopting, in a specific way, in order to automate calculations, of a method for linear determining of the ship’s oscillation parameters and of the sectional efforts in the ship's hull for dynamic layout on wave, on the basis of which the RLD-V1 program was developed. The validation of the method and program was done by comparing the results of the calculations obtained with the test measurements on 3 models. These tools have been a benchmark for the other methods and programs made, givend the large number of validations to which it has been subjected, and have proved to be particularly effective in the research work being carried out to establish proposals to improve the construction safety of the ship in in terms of longitudinal strength;
5. adopting, in a specific way, in order to automate calculations, of a method for nonlinear determining of the ship’s oscillation parameters and of the sectional efforts in the ship's hull for dynamic layout on wave, on the basis of which the RLD-V1N program was developed, considering linear damping depending on the ship’s speed. The validation of the method and program was done by comparing the results of the calculations obtained with the test measurements on 1 model. These tools allow a closer to reality assessment of the sectional efforts, ensuring the achievement of useful results in the research work carried out within the thesis;
6. development of an original method for nonlinear determination of the ship’s oscillation parameters and of sectional effort in the ship's hull at the dynamic layout on wave, on the basif of which the RLD-V2 program was developed, considering the nonlinear damping depending on the square of the ship’s speed. Taking into account the damping concept is a novelty in the study of vertical oscillations coupled with those of the ship's pitching, being an appreciation of the phenomenon closer to reality. Validation of the method and program was performed by comparing the results of the calculations obtained with the test measurements on 1 model. These tools allowed the closest to the reality assessment of the sectional efforts, ensuring the achievement of particularly interesting results in the research carried out in order to improve the safety of the ship's construction;
7. graphical display of the ship’s oscillations on waves and sectional efforts along the ship’s length, depending on time, by the aforementioned programs.

1. It follows from the above that, within the thesis, through an intensive study and research activity, a series of efficient calculating tools have been realized which have enabled achieving the purpose of the work to contribute by means of substantiated proposals to the improvement of the international and national regulations requirements on the construction safety of the ships.
2. It can be considered that the present thesis also contributes to a better knowledge of the complex hydrodynamic and strength phenomena that arise when ships navigate on waves or to a better probabilistic assessment of the safety of their construction under damage conditions, thus opening new perspectives for deepening of these areas for future research.

6.2 Original contributions

In order to achieve the proposed goal of the thesis, we made a series of original contributions of which more important were:

1. study on the current state of regulations concerning construction safety of ships and how they are being implemented from the design phase, continuing during construction, until the operational phase

2. adopting in a specific way, in order to automate calculations, of a method of determining the sectional efforts in the ship's hull for layout on still water and for the quasi-static layout on the cosine and trohoidal wave, as well as the elastic line of the hull
3. development of the RLS-V1 program based on the above mentioned method;
4. adopting, in a specific way, in order to automate calculations, of a method of linear determining the sectional efforts in the ship's hull for dynamic layout on wave;
5. development of the RLD-V1 program based on the above mentioned method;
6. adopting, in a specific way, in order to automate calculations, of a method of nonlinear determining of the sectional efforts in the ship's hull for dynamic layout on wave, considering linear damping depending on the ship's speed;
7. development of the RLD-V1N program based on the above mentioned method;
8. development of an original method of nonlinear determination of sectional effort in the ship's hull at the dynamic layout on wave, considering the nonlinear damping depending on the square of the ship's speed. The method can be generalized throughout the theory of oscillation and vibration mechanics;
9. development of the RLD-V2 program based on the above mentioned method
10. graphical display of the ship's oscillations on waves and sectional efforts along the ship's length, depending on time, by the aforementioned programs
11. resolving in a specific way of the nonlinear differential equation systems describing the ship's oscillations by the successive approximation method;
12. resolving in a specific way of the nonlinear differential equation systems describing the ship's oscillations by the β -Newmark method;
13. substantiated proposal to improve the construction safety of the ships by modifying of the IACS formulas to determine waves-induced sectional efforts;
14. proposal to improve the construction safety of the ships by probabilistic assessment of longitudinal residual strength of the ships when damaged;
15. proposal to improve the construction safety of the ships by probabilistic assessment of overall survival of the ships when damaged.

6.3 Future research perspectives

The present paper can be considered as a small bridge between the results of the research carried out so far and the next one, creating the perspective of approaching new themes such as:

1. verification of calculation methods and programs by making measurements on real ships;
2. research deepening to improve IACS formulas on longitudinal strength in order to allow building of the ships such as to ensure their safe operation;
3. completing the probabilistic analysis method of longitudinal strength of damaged seagoing ships with studies on the probability of locating the bottom damages

and establishing the assessment criteria according to [83];

4. completing the probabilistic analysis method of overall survival of damaged seagoing ships with studies on the probability of locating the bottom damages and establishing the assessment criteria according to [83];
5. improvement of the methods for determining the quadratic damping coefficients taking into account the complex configuration of the ship and the fact that they vary over time;
6. improvement of the mathematical methods of solving the systems of nonlinear differential equations in which both the system matrix and the free terms depend on the solution ;
7. expansion of the calculation methods and programs for dynamic ship’s layout on waves taking into account the slamming and hull’s vibration;
8. completion of the calculation methods and programs for dynamic ship’s layout on statistical waves according to various spectra, with fatigue analysis ;

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