DESCRIBING FUNCTION METHOD FOR PI-FUZZY CONTROLLED SYSTEMS STABILITY ANALYSIS

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Abstract: The paper proposes a global stability analysis method dedicated to fuzzy control systems containing Mamdani PI-fuzzy controllers with output integration to control SISO linear / linearized plants. The method is expressed in terms of relatively simple steps, and it is based on: the generalization of the describing function method for the considered fuzzy control systems to the MIMO case, the approximation of the describing functions by applying the least squares method. The method is applied to the stability analysis of a class of PI-fuzzy controlled servo-systems, and validated by considering a case study.

Keywords: fuzzy control, describing functions, stability analysis, PI controllers, servo-systems.

1. INTRODUCTION

Fuzzy control systems (abbreviated as FCSs) are nonlinear control systems, and the dynamic behaviour of these systems is more complex than that of linear systems. With this respect the main differences between the nonlinear systems and the linear ones are:
- the nonlinear systems can reach multiple steady-state regimes instead of the single-point attractor associated with the operating point in the case of linear systems,
- the nonlinear systems can perform long-term behaviours including the limit cycles and the chaotic behaviours, that are more complicated than the point attractors corresponding to linear systems.

In the general framework of the qualitative theory of nonlinear dynamical systems (for example, (Khalil, 1991)), several approaches and overviews (Driankov, *et al.*, 1993; Wang, 1997; Passino and Yurkovich, 1998; Sugeno, 1999) have been widely used for the stability analysis of FCSs. These approaches include:

(a) the state-space approach, based on a linearized model of the nonlinear dynamical system (Aracil, *et al.*, 1989; Garcia-Cerezo and Ollero, 1992; Precup, *et al.*, 2002),
(b) Popov's hyperstability theory (Opitz, 1993; Precup and Preitl, 1997),
(c) Lyapunov's stability theory (Passino and Yurkovich, 1998; Sugeno, 1999),
(d) the circle criterion (Driankov, *et al.*, 1993; Opitz, 1993; Passino and Yurkovich, 1998),
(e) the describing function method referred to also as the harmonic balance method (Kiendl, 1993; Passino and Yurkovich, 1998), etc.

This paper aims to propose a stability analysis method (SAM) belongs to the approach (e). It is dedicated to FCSs employing Mamdani PI-fuzzy controllers with output integration (PI-FC-OIs) to control SISO linear / linearized plants. The support for using a fuzzy controller (FC) to control a plant having a linear or linearized model is in the fact that this controlled plant (CP) model can be considered as a simplified model of a relatively complex model of the CP. So, the CP is nonlinear but linearized in the vicinity of a set of operating points or of a trajectory. The FC, as nonlinear element, can compensate – based on the designer's experience – the model uncertainties, nonlinearities and CP parametric variations.
The SAM proposed in the paper is based on the describing function method and represents a global SAM. Existing approaches to the describing function

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method used in the stability analysis of FCSs deal with the following applications of this method:

- in the form of the exponential-input describing function technique to FCSs using the representations of the FCs as multidimensional-multilevel relays (Abdelnour, *et al.*, 1993a, 1993b),
- to FCSs based on Mamdani FCs without dynamics (Kiendl, 1993; Passino and Yurkovich, 1998);
- to multivariable Takagi-Sugeno FCSs (Cuesta, *et al.*, 1999),
- to FCSs based on FCs with dynamics, by using the describing function of the saturation which occurs outside the universe of discourse region of the FCs (Aracil and Gordillo, 2000).

All these approaches require the prediction of the limit cycles, specific to the describing function method (Kim, *et al.*, 2000).

Since the PI-fuzzy controllers (PI-FCs) with dynamics can be considered from the point of view of the basic fuzzy controller (without dynamics) as MIMO systems, the main contribution of this paper is to generalize the describing function method to the MIMO case of FCSs based on Mamdani PI-FC-OIs. Applying the least squares method performs the approximation of the describing functions. The proposed SAM is expressed in terms of relatively simple to be proceeded steps.

This paper is organized as follow. In the next Section the Mamdani PI-FC-OIs it will be reviewed. In Section 3 the describing function method is applied, the resulting SAM is derived and its steps are presented. Then, Section 4 addresses the application of the SAM to the stability analysis of a class of FCSs meant for controlling a class of servo-systems and validates the proposed method by considering a case study. Section 5 draws the conclusions.

2. PI-FUZZY CONTROLLER WITH OUTPUT INTEGRATION

The structure of the considered FCS is a conventional one, presented in Fig. 1, where: r – the reference input, y – the controlled output, $e = r - y$ – the control error, u – the control signal, d_1, d_2, d_3 – the disturbance inputs, and the CP includes the actuator and the measuring device.

The considered PI-fuzzy controller with output integration (PI-FC-OI) can be considered as type-II fuzzy systems (Sugeno, 1999), or Mamdani FCs with singleton consequents, a special case of Takagi-Sugeno FCs. The PI-FC-OI represents a discrete-time FC with dynamics, introduced by:

- the numerical differentiation of control error e_k expressed as its increment, Δe_k, $\Delta e_k = e_k - e_{k-1}$,

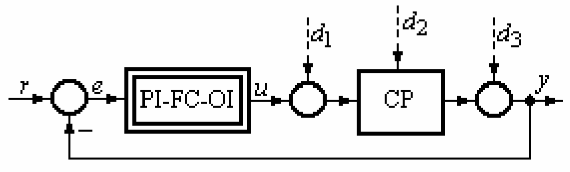

Fig. 1. Structure of fuzzy control system.

- the numerical integration of the increment of control signal, Δu_k, $u_k = u_{k-1} + \Delta u_k$,

with k – the index of current sampling interval. The PI-FC-OI structure is shown in Fig. 2, where B-FC represents the basic fuzzy controller, without dynamics.

The block B-FC is a nonlinear two inputs-single output (TISO) system, which includes among its nonlinearities the scaling of inputs and output as part of its fuzzification module. The fuzzification is solved in the initial phase by means of the regularly distributed input and output membership functions illustrated in Fig. 3. Other distributions of the membership functions can modify in a desired way the controller nonlinearities.

The inference engine in B-FC employs Mamdani's MAX-MIN compositional rule of inference assisted by the rule base presented in Table 1, and the centre of gravity method for singletons is used for defuzzification.

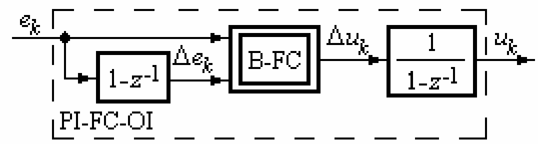

Fig. 2. Structure of PI-fuzzy controller with output integration.

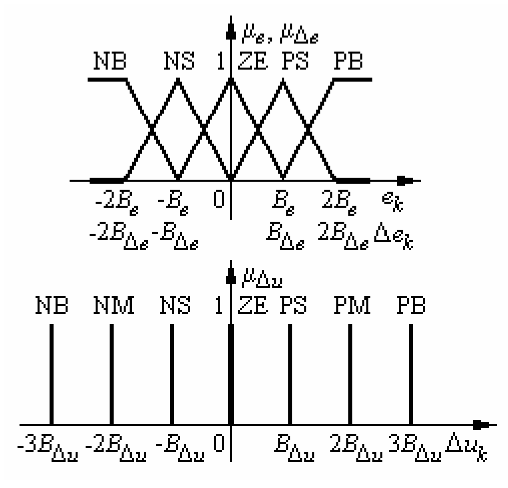

Fig. 3. Membership functions of input and output linguistic variables in B-FC.

Table 1 Decision table of B-FC

$\Delta e_k \backslash e_k$	NB	NS	ZE	PS	PB
PB	ZE	PS	PM	PB	PB
PS	NS	ZE	PS	PM	PB
ZE	NM	NS	ZE	PS	PM
NS	NB	NM	NS	ZE	PS
NB	NB	NB	NM	NS	ZE

Fig. 3 highlights the strictly positive parameters of the PI-FC-OI to be tuned by the development method, B_e, $B_{\Delta e}$ and $B_{\Delta u}$. For the development of the PI-FC-OI the start is in the expression of the discrete-time equation of a digital PI controller in its incremental version:

$$(1)\quad \Delta u_k = K_P \Delta e_k + K_I e_k = K_P (\Delta e_k + \alpha \cdot e_k).$$

In the case of a quasi-continuous digital PI controller the parameters K_P, K_I and α can be computed as functions of the parameters k_C (gain) and T_i (integral time constant) of a basic continuous-time PI controller having the transfer function (t.f.) $H_C(s)$:

$$(2)\quad H_C(s) = k_C (1 + 1/(sT_i)),$$

and the connections between $\{K_P, K_I, \alpha\}$ and $\{k_C, T_i\}$ have the following form when Tustin's method (the trapezoid rule) is employed:

$$(3)\quad \begin{aligned} &K_P = k_C[1 - T_s/(2T_i)],\ K_I = k_C T_s / T_i, \\ &\alpha = K_I / K_P = 2T_s/(2T_i - T_s), \end{aligned}$$

with T_s – sampling period chosen in accordance with the requirements of quasi-continuous digital control.

The design relations of the PI-FC-OI are obtained by the application of the modal equivalence principle (Galichet and Foulloy, 1995) particularized as (4):

$$(4)\quad B_{\Delta e} = \alpha \cdot B_e,\ B_{\Delta u} = K_I B_e,$$

where the free parameter B_e represents the designer's option. Using the experience in controlling the plant one can choose the value of this parameter, but firstly it must be chosen to ensure the aim of a stable FCS.

3. DESCRIBING FUNCTION METHOD

For the formulation of the SAM based on the application of the describing function method it is necessary to transform the initial FCS structure into a MIMO one because the block B-FC in Fig. 2 is a TISO system. This modified FCS structure is illustrated in Fig. 4, where the dynamics of the fuzzy controller (its linearized part) is transferred to the plant CP resulting in the extended controlled plant (ECP, a linear block). The vectors in Fig. 4, $\mathbf{r}_k$ – the reference input vector, $\mathbf{e}_k$ – the control error vector, $\mathbf{u}_k$ – the control signal vector and $\mathbf{y}_k$ – the controlled output vector, are defined as follows:

$$(5)\quad \begin{aligned} &\mathbf{r}_k = [r_k\ \ \Delta r_k]^T,\ \mathbf{e}_k = [e_k\ \ \Delta e_k]^T, \\ &\mathbf{u}_k = [\Delta u_k\ \ \Delta u_{f,k}]^T,\ \mathbf{y}_k = [y_k\ \ \Delta y_k]^T, \end{aligned}$$

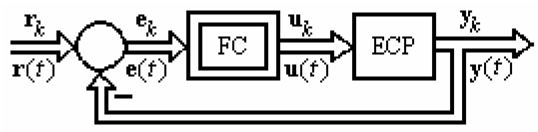

Fig. 4. Structure of FCS used in stability analysis.

where $\Delta \mathbf{v}_k = \mathbf{v}_k - \mathbf{v}_{k-1}$ stands for the increment of the vector $\mathbf{v}_k$, $\Delta u_{f,k}$ is the fictitious increment of control signal and the upper index T stands for matrix transposition.

The FCS structure in Fig. 4 can be used in both the discrete-time case and the continuous-time case, with the vectors defined in terms of (6):

$$(6)\quad \begin{aligned} &\mathbf{r}(t) = [w_1(t)\ \ w_2(t)]^T,\ \mathbf{e}(t) = [i_1(t)\ \ i_2(t)]^T, \\ &\mathbf{u}(t) = [c_1(t)\ \ c_2(t)]^T,\ \mathbf{y}(t) = [o_1(t)\ \ o_2(t)]^T, \end{aligned}$$

and the correspondence between the vectors in (5) and (6) is immediate.

The nonlinear input-output static map **F** characterizes the block FC in Fig. 4:

$$(7)\quad \mathbf{F}: R^2 \to R^2,\ \mathbf{F}(\mathbf{e}_k) = [f(\mathbf{e}_k),\ 0]^T,$$

where f ($f: R^2 \to R$) is the input-output static map of the nonlinear TISO system B-FC in Fig. 2. Therefore, the modified FCS structure presented in Fig. 4 has only one time-invariant and memoryless nonlinear component, the block FC.

In this context, for the application of the describing function method the following two conditions must be fulfilled (Passino and Yurkovich, 1998):
- the nonlinear function **F** should be an odd function;
- the linear block ECP should have characteristics of a low-pass filter.

All variables in the modified FCS structure, presented in Fig. 4, have two components (see (5) and (6)). This required the introduction of a fictitious control signal, supplementary to the outputs of the block B-FC, for obtaining an equal number of inputs and outputs due to the necessity of the stability theory in the MIMO case (Landau, 1979).

The mathematical model of the block ECP can be derived by starting with the t.f. of the controlled plant with respect to the control signal (see Fig. 1), $H_{CP}(s)$. Discetizing $H_{CP}(s)$ by taking into account the presence of a zero-order hold – in the conditions of a sampling period chosen in accordance with the requirements of quasi-continuous digital control – leads to the pulse t.f. of the controlled plant, $H_{ZCP}(z)$:

$$(8)\quad H_{ZCP}(z) = (1 - z^{-1}) Z\{L^{-1}[H_{CP}(s)/s]\}.$$

The pulse t.f. of the CP together with the integrator (in Fig. 2), $H_{ICP}(z)$, can be expressed as:

$$(9)\quad H_{ICP}(z) = Z\{L^{-1}[H_{CP}(s)/s]\}.$$

For obtaining continuous-time equivalents $H_{11}(s)$ and $H_{21}(s)$ of the pulse t.f.s in (9) and (8), respectively, it is applied Tustin's method that maps the z-plane to the s-plane (Franklin, *et al.*, 1998):

$$H_{11}(s) = H_{ICP}(z)\Big|_{z=\frac{1+T_s/2}{1-T_s/2}}, \quad H_{21}(s) = H_{ZCP}(z)\Big|_{z=\frac{1+T_s/2}{1-T_s/2}}. \qquad (10)$$

By using the definition of the vectors in (6), the transfer matrix $\mathbf{H}(s)$ of the block ECP will have the expression (11):

$$(11) \quad \mathbf{H}(s) = [H_{ij}(s)]_{i,j=\overline{1,2}}.$$

The two t.f.s $H_{12}(s)$ and $H_{22}(s)$ in (11) have no effect on the FCS stability analysis because they correspond to the fictitious control signal $c_2(t)$ (in the discrete case, the fictitious increment of control signal $\Delta u_{f,k}$). This is also the reason why the second component of $\mathbf{F}$ in (6) is zero.

For an input $\mathbf{e}(t)$ (defined in (6)) to the block FC (in Fig. 4):

$$(12) \quad i_l(t) = A_l \sin(\omega t),\ l = \overline{1,2},$$

where the input amplitudes are $\mathbf{a} = [A_1 \ \ A_2]^T$, $A_l > 0$, $l = \overline{1,2}$, and the frequency is $\omega > 0$, there will be obtained the output $\mathbf{u}(t)$ (defined in (6)) expanded into a Fourier series:

$$(13) \quad c_m(t) = a_{0,m}/2 + \sum_{n=1}^{\infty}[a_{n,m}\cos(n\omega t) + b_{n,m}\sin(n\omega t)],\ m = \overline{1,2},$$

with the following expressions of the Fourier coefficients:

$$(14) \quad a_{0,m} = (1/\pi)\int_{-\pi}^{\pi} c_m(t)d(\omega t),\ a_{n,m} + jb_{n,m} = (1/\pi)\int_{-\pi}^{\pi} c_m(t)e^{j\omega t}d(\omega t),\ n \in N^*,\ m = \overline{1,2}.$$

In the accepted conditions, $a_{0,m} = 0$ and by neglecting the higher order harmonics with respect to the fundamental harmonics, the result will be expressed as the describing functions $N_{ml}(\mathbf{a}, \omega)$ that will not depend on ω and so will be written as $N_{ml}(\mathbf{a})$:

$$(15) \quad N_{ml}(\mathbf{a}) = b_{1,m}/A_l,\ l,m = \overline{1,2}.$$

Neglecting the effect of the fictitious (increment of) control signal on the stability analysis requires the necessity for:

$$(16) \quad N_{2l}(\mathbf{a}) = 0,\ l = \overline{1,2}.$$

By defining the describing function matrix $\mathbf{N}(\mathbf{a})$:

$$(17) \quad \mathbf{N}(\mathbf{a}) = [N_{ml}(\mathbf{a})]_{l,m=\overline{1,2}},$$

the harmonic balance equation can be written as (Cuesta, *et al.*, 1999):

$$(18) \quad (\mathbf{H}(j\omega)\mathbf{N}(\mathbf{a}) + \mathbf{I})\mathbf{a} = [0 \ \ 0]^T,\ \mathbf{I} = \mathrm{diag}(1,1).$$

If any limit cycles exist in the FCS, then their existence can be predicted by solving the equation (18) with respect to $\mathbf{a}$ and ω.

There can be employed several ways for the computation of the describing functions. This problem can be solved easily if the analytical expression of the function f is known. If this is not the case, the problem is more complex and can be solved by using the numerical integration in (14). Another approach is based on performing an experiment with the nonlinear block FC in Fig. 4, assisted by the application of the least squares method. Similar experiments were performed by Passino and Yurkovich (1998) but in the SISO case.

The SAM can be formulated in terms of the following steps:

- Step 1: Express the transfer matrix of the block ECP in (11) by using (8) … (10).

- Step 2: Perform the experiments and apply the least squares method to compute a sufficient number M of values of the describing function matrix $\mathbf{N}(\mathbf{a}^q)$, $q = \overline{1,M}$.

- Step 3: Obtain the solutions of the harmonic balance equation (18) by using numerical techniques to solve the optimization problem (19):

$$(19) \quad \min_{\mathbf{a}\in R^2,\ \omega>0} \|(\mathbf{H}(j\omega)\mathbf{N}(\mathbf{a}) + \mathbf{I})\mathbf{a}\|,$$

having the solutions $\mathbf{a}^*$ – the input amplitudes and ω^* – the frequency of the limit cycles, and $\|\mathbf{v}\|$ represents generally the Euclidean norm of the vector $\mathbf{v}$.

- Step 4: Solve the optimization problem (20):

$$(20) \quad \min_{\mathbf{a}\in R^2,\ \omega>0} \|(\mathbf{H}(j\omega)\mathbf{N}(\mathbf{a}) + \mathbf{I})\mathbf{a} - \boldsymbol{\varepsilon}_i\|,$$

for two relatively small values of $\boldsymbol{\varepsilon}_i = [\varepsilon_{1,i} \ \ \varepsilon_{2,i}]^T$, $i = \overline{1,2}$, $\varepsilon_{1,1} > 0,\ \varepsilon_{2,1} > 0,\ \varepsilon_{1,2} < 0,\ \varepsilon_{2,2} < 0$, and the solutions of (20) are $\mathbf{a}_i$ and ω_i, $i = \overline{1,2}$.

- Step 5: If $\|\mathbf{a}_1\| < \|\mathbf{a}^*\|$ and $\|\mathbf{a}_2\| > \|\mathbf{a}^*\|$, then the corresponding limit cycle is stable and the FCS is globally stable. The FCS is globally stable also if the equation (19) does not have any solutions.

4. APPLICATION

This Section is dedicated to the validation of the proposed SAM by its application to the stability analysis of a class of FCSs dedicated to control of a class of servo-systems. In this case the CP is characterized by the t.f. (21):

$$(21)\quad H_{CP}(s) = k_P / [(1 + sT_\Sigma)(1 + sT_1)]\,,$$

where: T_1 – large time constant, T_Σ – small time constant or time constant corresponding to the sum of parasitic time constants ($T_\Sigma << T_1$), and k_P – gain. In these conditions, from a theoretical point of view, $H_{CP}(s)$ has a quasi-integral behaviour, and the benchmark (22) can be used to approximate (21):

$$(22)\quad H_{CP}(s) = k_P / [s(1 + sT_\Sigma)]\,.$$

For the CP (22) the use of PI controllers having the t.f. (2) or (23) used as basic continuous-time PI controllers in accordance with Section 2:

$$(23)\quad H_C(s) = k_c(1 + sT_i)/s,\ k_c = k_C / T_i\,,$$

with the gain k_c, tuned in terms of the Extended Symmetrical Optimum (ESO) method (Preitl and Precup, 1999), can ensure good control system (CS) performance:

$$(24)\quad k_c = 1/(\sqrt{\beta^3}T_\Sigma^2 k_P),\ T_i = \beta T_\Sigma,\ k_C = k_c T_i\,,$$

where β represents a single design parameter.

The tuning relations (24) were obtained by applying the optimization conditions (25) specific to the ESO method:

$$(25)\quad \sqrt{\beta}a_0 a_2 = a_1^2,\ \sqrt{\beta}a_1 a_3 = a_2^2\,,$$

to the closed-loop t.f. with respect to the reference input $H_r(s)$ (it is considered the CS structure in Fig. 1, with the continuous-time PI controller instead of the PI-FC-OI):

$$(26)\quad H_r(s) = (b_0 + b_1 s)/(a_0 + a_1 s + a_2 s^2 + a_3 s^3),\ b_0 = a_0,\ b_1 = a_1\,.$$

In the case of the controlled plant (19) and the PI controller with the t.f. (23), the coefficients of $H_r(s)$ in (26) can be expressed in terms of (27):

$$(27)\quad a_0 = k_c k_P,\ a_1 = k_c k_P T_1,\ a_2 = T_1 + T_\Sigma,\ a_3 = T_1 T_\Sigma\,.$$

Applying the optimization conditions (25) and introducing the following notation specific to the CP:

$$(28)\quad m_P = T_\Sigma / T_1 << 1\,,$$

the tuning relations will obtain the form (29) in this case corresponding to an extension of the ESO method (Preitl, *et al.*, 2002):

$$(29)\quad \begin{aligned} &k_c = (1 + m_P)^3 / (\sqrt{\beta^3}T_\Sigma k_P m_P),\ T_i = \beta T_{\Sigma m}\,,\\ &T_{\Sigma m} = T_\Sigma[m_P^2 + (2 - \sqrt{2})m_P + 1]/(1 + m_P)^3\,. \end{aligned}$$

The recommended values for β are again β ∈ (1, 20). The considered case study deals with a servo-system having the t.f. of the CP in its simplified form (21), with the parameters $k_P = 1$, $T_\Sigma = 1$ s and $T_1 = 10$ s, that fulfil the condition (28) because $m_P = 0.1$.

The development of the PI-FC-OI is performed according to Section 2, but it starts with the development of the basic continuous-time PI controller in terms of this Section.

Choosing β = 9 and applying (29), the PI controller parameters will have the values k_c = 0.493, T_i = 7.2256 s, k_C = 3.5619. Setting T_s = 0.5 s, the parameters of the quasi-continuous digital PI controller result from (3): K_P = 3.5273, K_I = 0.2465, α = 0.0699. The choice of the PI-FC-OI parameter B_e = 0.5 and (4) will lead to the other two PI-FC-OI parameters, $B_{\Delta e}$ = 0.0349 and $B_{\Delta u}$ = 0.1232.

For the developed PI-FC-OI and the accepted CP, there will be applied as follows the steps 1 … 5 of the SAM according to the previous Section.

- Step 1: The useful elements of the transfer matrix $\mathbf{H}(s)$ are:

$$(30)\quad \begin{aligned} &H_{11}(s) = B_{11}(s)/A_{11}(s),\ B_{11}(s) = 0.0003s^3 +\\ &+ 0.0145s^2 + 0.1024s + 0.1959\,,\\ &A_{11}(s) = s(s^2 + 1.0797s + 0.0979)\,,\\ &H_{21}(s) = B_{21}(s)/A_{21}(s)\,,\\ &B_{21}(s) = 0.0006s^2 + 0.02767s + 0.0979\,,\\ &A_{21}(s) = s^2 + 1.0797s + 0.0979\ ; \end{aligned}$$

- Step 2: Due to the distribution of the membership functions (Fig. 3) and to the defuzzification method employed, the PI-FC-OI will have quasi-PI behaviour (that means it will be almost linear), and it is sufficient a relatively small of values of the describing function matrix, M = 10. In the case ω = 1 rad/s, the elements of $\mathbf{N}(\mathbf{a}^q)$ are:

$$(31)\quad \begin{aligned} &\mathbf{a}^q(s) \in \{0.1, 0.2, 0.4, 0.5, 0.6, 0.8, 1, 1.1,\\ &1.2, 1.5\},\ N_{11}^q = N_{12}^q \in \{0.308, 0.232, 0.2377,\\ &0.2332, 0.2468, 0.246, 0.2437, 0.2408,\\ &0.2307, 0.2437\},\\ &N_{11}^q = N_{12}^q = 0,\ q = \overline{1,10}\,. \end{aligned}$$

It can be observed that some elements in (31) are equal one to another because the nonlinearity of the block B-FC is symmetrical (because of the membership functions shapes in Fig. 3 and of the decision table in Table 1). Other elements in (31) are zero to neglect the effect of the fictitious increment of control signal, $\Delta u_{f,k}$.

- Step 3: The equation (19) does not have any solutions, which ensures that the FCS is globally stable in the case $B_e = 0.5$. This is the reason why the steps 4 and 5 are no more necessary.

The behaviour of the considered FCS is illustrated in Fig. 5.

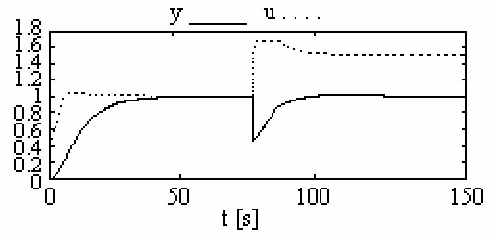

Fig. 5. Behaviour of fuzzy control system.

The simulation conditions are characterized by the unit step modification of r followed by a −0.5 step modification of the disturbance input d_3 (after 75 s), where the continuous line is used for y and the dotted line for u. This behaviour proves that the FCS is globally stable for the developed FC.

5. CONCLUSIONS

The proposed SAM represents a generalization to the MIMO case of other limit cycle criteria. It provides useful relations to the choice of the free parameter of the PI-FC-OI, B_e.

The SAM enables the redesign of the fuzzy controllers to avoid the existence of limit cycles, and it can be also considered as part of a development for PI fuzzy controllers.

The application presented in the paper validates the proposed SAM and opens perspectives for its application to other structures of fuzzy controllers with dynamics.

Although the plant has been assumed linear, it must be seen as a linearized one, corresponding to a more complex model of the CP. This can be considered because generally the goal of fuzzy control is not to control simple plants but to initially approach as a convenient and understandable nonlinear solution the control of a complex, uncertain or not well-defined plant.

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