Annals of "Dunarea de Jos" University of Galati
Fascicle I. Economics and Applied Informatics
Years XXI – n°2/2015
ISSN-L 1584-0409 ISSN-Online 2344-441X
www.eia.feaa.ugal.ro

Statistical Modeling and Analysis of the Correlation between the Gross Domestic Product per Capita and the Life Expectancy

Gabriela OPAIT *

ARTICLE INFO

Article history:
Accepted May 2014
Available online 14 August 2014
JEL Classification
C 1, C 12, C 2

Keywords:
Gross Domestic Product per capita, Life Expectancy, Least Squares Method

ABSTRACT

This paper reflects the statistical modeling of the trends concerning the G.D.P. per capita and the Life Expectancy in 10 countries, in 2014, respectively the G.D.P. per capita and the Life Expectancy in Romania, between 2010-2014. With the help of the „Least Squares Method", which is a method through we can to reflect the trend line of the best fit concerning a model. This research reflects that there is a strong intensity regarding the correlation between the G.D.P. per capita and the Life Expectancy in the world. So, if everyone's needs of the people are satisfied, then any increase in G.D.P. per capita would barely affect life expectancy.

1. Introduction

In this research, I present a personal contribution which reflects a statistical analysis of the trends models between the Gross Domestic Product per capita and the Life Expectancy in 10 countries, in 2014, respectively the Gross Domestic Product per capita and the Life Expectancy in Romania, in the period 2010-2014. The purpose of the research reflects the possibility for to express the intensity of the correlation with the help of the Pearson correlation coefficient. The statistical methods used are the „Coefficients of Variation Method", respectively the „Least Squares Method" applied for to calculate the parameters of the regression equation and the Pearson Method of Correlation used for to reflect the intensity of the correlation between the G.D.P. per capita and the Life Expectancy. The sections 2, respectively the section 4, present the methodology for to achieve the trends models between the G.D.P. per capita and the Life Expectancy in 10 countries, in 2014, respectively the G.D.P. per capita and the Life Expectancy in Romania, in the period 2010-2014, with the help of the „Least Squares Method". The section 3, respectively the section 5, reflect the intensity of the correlation between the G.D.P. per capita and the Life Expectancy, in 10 countries, respectively in Romania. The state of the art in this domain is represented by the research belongs to Carl Friederich Gauss, who created the „Least Squares Method" and Karl Pearson, who achieved the Pearson correlation coefficient [1], [3].

2. The modeling of the trend between the G.D.P. per capita and the Life Expectancy in 2014, for 10 countries

In 2014, we observe the next evolution concerning the G.D.P. per capita and the Life Expectancy in 10 countries, according to the table no. 1:

Table 1 The evolution of the G.D.P. per capita and the Life Expectancy in 2014

COUNTRIES	GROSS DOMESTIC PRODUCT PER CAPITA IN 2014, ($)	LIFE EXPECTANCY 2014
ITALY	35.823	82,63
JAPAN	36.332	84,46
FRANCE	44.538	81,66
UNITED KINGDOM	45.653	80,42
GERMANY	47.590	80,44
AUSTRIA	51.307	80,17
NETHERLANDS	51.373	81,12
U.S.A.	54.597	79,56
SWEDEN	58.491	81,89
DENMARK	60.564	79,09

Source: EUROSTAT

*Faculty of Economics and Business Administration, Dunarea de Jos University of Galati, Romania. E-mail address: gopait@ugal.ro (G. Opait)

We want to identify the trend model between the G.D.P. per capita and the Life Expectancy for these 10 countries in 2014 , using the table no.1.

- if we formulate the null hypothesis H_0 *:* which mentions the assumption of the existence for the model of tendency concerning *Y* factor, where *Y = the Life Expectancy,* as being the function $y_{t_i} = a + b \cdot x_i$, then the parameters *a* and *b* of the adjusted linear function, can to be calculated by means of the next system [1]:

$$\begin{cases} n \cdot a + b\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i \\ a\sum_{i=1}^{n} x_i + b \cdot \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i \cdot y_i \end{cases}$$

Therefore, $a = \dfrac{\sum_{i=1}^{n} x_i^2 \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_i y_i}{n\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i^2)}$ $\quad b = \dfrac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i^2)}$

Table 2 The estimate of the value for the variation coefficient in the case of the adjusted linear function, in the hypothesis concerning the linear evolution of the correlation between the G.D.P. per capita and the Life Expectancy in 10 countries, in 2014

COUNTRIES	G.D.P. per capita ($) 2014 (x_i)	LIFE EXPECTANCY 2014 (y_i)	*LINEAR TREND*			
			x_i^2	$x_i y_i$	$y_{x_i} = a + bx_i$	$\lvert y_i - y_{x_i} \rvert$
ITALY	**35.823**	**82,63**	1283287329	2960054,49	82,9335005504	0,30
JAPAN	**36.332**	**84,46**	1320014224	3068600,72	82,8623611325	1,60
FRANCE	**44.538**	**81,66**	1983633444	3636973,08	81,7154651339	0,06
UNITED KINGDOM	**45.653**	**80,42**	2084196409	3671414,26	81,5596292774	1,14
GERMANY	**47.590**	**80,44**	2264808100	3828139,60	81,2889081527	0,85
AUSTRIA	**51.307**	**80,17**	2632408249	4113282,19	80,7694087100	0,60
NETHERLANDS	**51.373**	**81,12**	2639185129	4167377,76	80,7601843454	0,36
U.S.A.	**54.597**	**79,56**	2980832409	4343737,32	80,3095881110	0,75
SWEDEN	**58.491**	**81,89**	3421197081	4789827,99	79,7653505996	2,12
DENMARK	**60.564**	**79,09**	3667998096	4790006,76	79,4756216933	0,39
TOTAL	**486268**	**811,44**	24277557470	39369414,17	811,4400177062	8,17

If we calculate the statistical data for to adjust the linear function, we obtain for the parameters *a* and *b* the values:

$$a = \frac{24.277.557.470 \cdot 811{,}44 - 486.268 \cdot 39.369.414{,}17}{10 \cdot 24.277.557.470 - (486.268)^2} = 87{,}9402340817$$

$$b = \frac{10 \cdot 39.369.414{,}17 - 486.268 \cdot 811{,}44}{10 \cdot 24.277.577.470 - (486.268)^2} = -0{,}0001397631$$

Hence, the coefficient of variation for the adjusted linear function is:

$$v_I = \left[\frac{\sum_{i=-m}^{m} \left| y_i - y_{t_i}^I \right|}{n} : \frac{\sum_{i=-m}^{m} y_i}{n} \right] \cdot 100 = \frac{\sum_{i=-m}^{m} \left| y_i - y_{t_i}^I \right|}{\sum_{i=-m}^{m} y_i} \cdot 100 = \frac{8{,}17}{811{,}44} \cdot 100 = 1{,}007\%$$

- in the situation of the alternative hypothesis H_1 *:* which specifies the assumption of the existence for the model of tendency regarding *Y* factor, where *Y = the Life Expectancy*, as being the quadratic function $y_{t_i} = a + b \cdot x_i + cx_i^2$, the parameters *a*, *b* şi *c* of the adjusted quadratic function, can to be calculated by means of the system [1]:

$$S = \sum_{i=1}^{n} (y_i - y_{ti})^2 = \min \Leftrightarrow S = \sum_{i=1}^{n} (y_i - a - bx_i - cx_i^2)^2 = \min$$

$$\begin{cases} \dfrac{\partial S}{\partial a} = 0 \\ \dfrac{\partial S}{\partial b} = 0 \\ \dfrac{\partial S}{\partial c} = 0 \end{cases} \Rightarrow \begin{cases} 2\sum_{1=1}^{n} (y_i - a - bx_i - cx_i^2)(-1) = 0 / (-\frac{1}{2}) \\ 2\sum_{1=1}^{n} (y_i - a - bx_i - cx_i^2)(-x_i) = 0 / (-\frac{1}{2}) \\ 2\sum (y_i - a - bx_i - cx_i^2)(-x_i^2) = 0 / (-\frac{1}{2}) \end{cases} \Rightarrow$$

Therefore,

$$\begin{cases} n \cdot a + b\sum_{i=1}^{n} x_i + c\sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} y_i \\ a\sum_{i=1}^{n} x_i + b \cdot \sum_{i=1}^{n} x_i^2 + c\sum_{i=1}^{n} x_i^3 = \sum_{i=1}^{n} x_i \cdot y_i \\ a \cdot \sum_{i=1}^{n} x_i^2 + b\sum_{i=1}^{n} x_i^3 + c\sum_{i=1}^{n} x_i^4 = \sum_{i=1}^{n} x_i^2 \cdot y_i \end{cases}$$

Table 3 The estimates of the value for the variation coefficient in the case of the adjusted quadratic function, in the hypothesis concerning the parabolic evolution of the correlation between the G.D.P. per capita and the Life Expectancy in 10 countries, in 2014

COUN-TRIES	G.D.P. per capita ($) 2014 ($x_i$)	Life Expectancy 2014 (y_i)	*PARABOLIC TREND*					
			x_i^2	x_i^3	x_i^4	$x_i^2 y_i$	$y_{t_i} = a + bx_i + cx_i$	$\|y_i - y_{t_i}$
ITALY	**35,823**	**82,63**	1283287329	45971,20 1986767	1646826,36 87719	106038,031 99527	57,7240669	24,91
JAPAN	**36,332**	**84,46**	1320014224	47958,75 6786368	1742437,55 15623	111488,401 35904	61,50188399	22,96
FRANCE	**44,538**	**81,66**	1983633444	88347,06 6328872	3934801,64 01553	161983,507 03704	99,19317673	17,53
UNITED KINGDOM	**45,653**	**80,42**	2084196409	95149,81 8660077	4343874,67 12884	167611,075 21178	100,9409722	20,52
GERMANY	**47,590**	**80,44**	2264808100	107782,2 17479	5129355,72 98256	182181,163 564	102,0583242	21,62
AUSTRIA	**51.307**	**80,17**	2632408249	135060,9 7003144	6929573,18 94032	211040,169 32233	97,38066949	17,21
NETHERLANDS	**51,373**	**81,12**	2639185129	135582,8 5763211	6965298,14 51347	214090,697 66448	97,21656661	16,1
U.S.A.	**54,597**	**79,56**	2980832409	162744,5 0703417	8885361,85 05447	224123,814 50004	85,75733026	6,2
SWEDEN	**58,491**	**81,89**	3421197081	200109,2 3846477	11704589,4 67042	280161,828 96309	62,9195437	18,97
DENMARK	**60,564**	**79,09**	3667998096	222148,6 3668614	13454210,0 32259	290101,969 41264	46,74649181	32,34
TOTAL	**486,268**	**811,44**	24277557470	1240855, 2710897	64736328,6 45987	1948820,65 90297	811,4390259	198,35

If we calculate the statistical data for to adjust the quadratic function, we obtain for the parameters *a, b* and *c* the next values:

$$\begin{cases} 10\cdot a + 486{,}268\cdot b + 24277{,}557470\cdot c = 811{,}44 \\ 486{,}268\cdot a + 24277{,}557470\cdot b + 1240855{,}2710897\cdot c = 39369{,}41417 \\ 24277{,}557470\cdot a + 1240855{,}2710897\cdot b + 64736328{,}645987\cdot c = 1948820{,}6590297 \end{cases} \Rightarrow$$

$$a = -630{,}6310642 \qquad b = 30{,}84364327 \qquad c = -0{,}324601274$$

So, the coefficient of variation for the adjusted quadratic function has the value:

$$v_{II} = \left[\frac{\sum_{i=-m}^{m} \left| y_i - y_{t_i}^{II} \right|}{n} : \frac{\sum_{i=-m}^{m} y_i}{n} \right] \cdot 100 = \frac{\sum_{i=-m}^{m} \left| y_i - y_{t_i}^{II} \right|}{\sum_{i=-m}^{m} y_i} \cdot 100 = \frac{198{,}35}{811{,}44} \cdot 100 = 24{,}44\%$$

- in the case of the alternative hypothesis H_2 *:* which describes the supposition of the existence for the model of tendency concerning *Y* factor, where *Y = the Life Expectancy*, as being the exponential function $y_{t_i} = ab^{x_i}$, then the parameters *a* and *b* of the adjusted exponential function, can to be calculated by means of the next system [1]:

$$S = \sum_{i=1}^{n} (\lg y_i - \lg y_{t_i})^2 = \min \Leftrightarrow S = \sum_{i=1}^{n} (\lg y_i - \lg a - x_i \lg b)^2 = \min$$

$$\begin{cases} \dfrac{\partial S}{\partial \lg a} = 0 \\ \dfrac{\partial S}{\partial \lg b} = 0 \end{cases} \Rightarrow \begin{cases} 2\sum_{1=1}^{n} (\lg y_i - \lg a - x_i \lg b)(-1) = 0 /(-\frac{1}{2}) \\ 2\sum_{1=1}^{n} (\lg y_i - \lg a - x_i \lg b)(-x_i) = 0 /(-\frac{1}{2}) \end{cases} \Rightarrow$$

$$\begin{cases} n\cdot \lg a + \lg b \cdot \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} \lg y_i \\ \lg a \sum_{i=1}^{n} x_i + \lg b \cdot \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i \cdot \lg y_i \end{cases}$$

Thus,

$$\lg a = \frac{\begin{vmatrix} \sum_{i=1}^{n} \lg y_i & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i \lg y_i & \sum_{i=1}^{n} x_i^{\,2} \end{vmatrix}}{\begin{vmatrix} n & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^{\,2} \end{vmatrix}} = \frac{\sum_{i=1}^{n} \lg y_i \sum_{i=1}^{n} x_i^{\,2} - \sum_{i=1}^{n} x_i \lg y_i \sum_{i=1}^{n} x_i}{n\sum_{i=1}^{n} x_i^{\,2} - \left(\sum_{i=1}^{n} x_i \right)^2}$$

and

$$\lg b = \frac{\begin{vmatrix} n & \sum_{i=1}^{n} \lg y_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i \lg y_i \end{vmatrix}}{\begin{vmatrix} n & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^{\,2} \end{vmatrix}} = \frac{n \cdot \sum_{i=1}^{n} x_i \lg y_i - \sum_{i=1}^{n} \lg y_i \sum_{i=1}^{n} x_i}{n\sum_{i=1}^{n} x_i^{\,2} - \left(\sum_{i=1}^{n} x_i \right)^2}$$

Table 4. The estimate of the value for the variation coefficient in the case of the adjusted exponential function, in the hypothesis concerning the exponential evolution of the correlation between the G.D.P. per capita and the Life Expectancy in 10 countries in 2014

COUN-TRIES	G.D.P. per capita ($) 2014 ($x_i$)	Life Expectancy 2014 (y_i)	*EXPONENTIAL TREND*				
			$\lg y_i$	$x_i \lg y_i$	$\lg y_{xi} = = \lg a + x_i \lg b$	$y_{xi} = ab^{x_i}$	$\|y_i - y_{t_i}\|$
ITALY	**35.823**	**82,63**	1,917137753	68677,62572	1,9187017763	82,92811161	0,3
JAPAN	**36.332**	**84,46**	1,926651077	69999,08693	1,9183233857	82,85588968	1,6
FRANCE	**44.538**	**81,66**	1,912009376	85157,07357	1,9122230453	81,70018595	0,04
UNITED KINGDOM	**45.653**	**80,42**	1,905364069	86985,58583	1,9113941543	81,54440234	1,12
GERMANY	**47.590**	**80,44**	1,905472062	90681,41543	1,9099541885	81,27447784	0,83
AUSTRIA	**51.307**	**80,17**	1,904011884	97689,13771	1,9071909707	80,75900695	0,59
NETHERLANDS	**51.373**	**81,12**	1,909127942	98077,62976	1,9835232827	96,27716246	15,16
U.S.A.	**54.597**	**79,56**	1,900694774	103772,2326	1,9859200043	96,80995182	17,55
SWEDEN	**58.491**	**81,89**	1,913230871	111906,7869	1,9018503851	79,77198244	2,12
DENMARK	**60.564**	**79,09**	1,898121576	114957,8351	1,9003093169	79,48941778	0,4
TOTAL	**486268**	**811,44**	19,09182141	927904,4096			39,71

Consequently, if we calculate the statistical data for to adjust the exponential function, we obtain for the parameters *a* and *b* the values:

$$\lg a = \frac{\begin{vmatrix} 19{,}09182141 & 486268 \\ 927904{,}4096 & 24277557470 \end{vmatrix}}{\begin{vmatrix} 10 & 486268 \\ 486268 & 24277557470 \end{vmatrix}} = 1{,}9453325945$$

$$\lg b = \frac{\begin{vmatrix} 10 & 19{,}09182141 \\ 486268 & 927904{,}4096 \end{vmatrix}}{\begin{vmatrix} 10 & 486268 \\ 486268 & 24277557470 \end{vmatrix}} = -0{,}0000007434$$

Accordingly, the coefficient of variation for the adjusted exponential function has the next value:

$$v_{\exp} = \left[\frac{\sum_{i=-m}^{m} \left| y_i - y_{t_i}^{\exp} \right|}{n} : \frac{\sum_{i=-m}^{m} y_i}{n} \right] \cdot 100 = \frac{\sum_{i=-m}^{m} \left| y_i - y_{t_i}^{\exp} \right|}{\sum_{i=-m}^{m} y_i} \cdot 100 = \frac{39{,}71}{811{,}44} \cdot 100 = 4{,}89\%$$

We apply the coefficients of variation method as criterion of selection for the best model of trend. We notice that:

$$v_I = 1{,}01\% < v_{\exp} = 4{,}89\% < v_{II} = 24{,}44\%$$

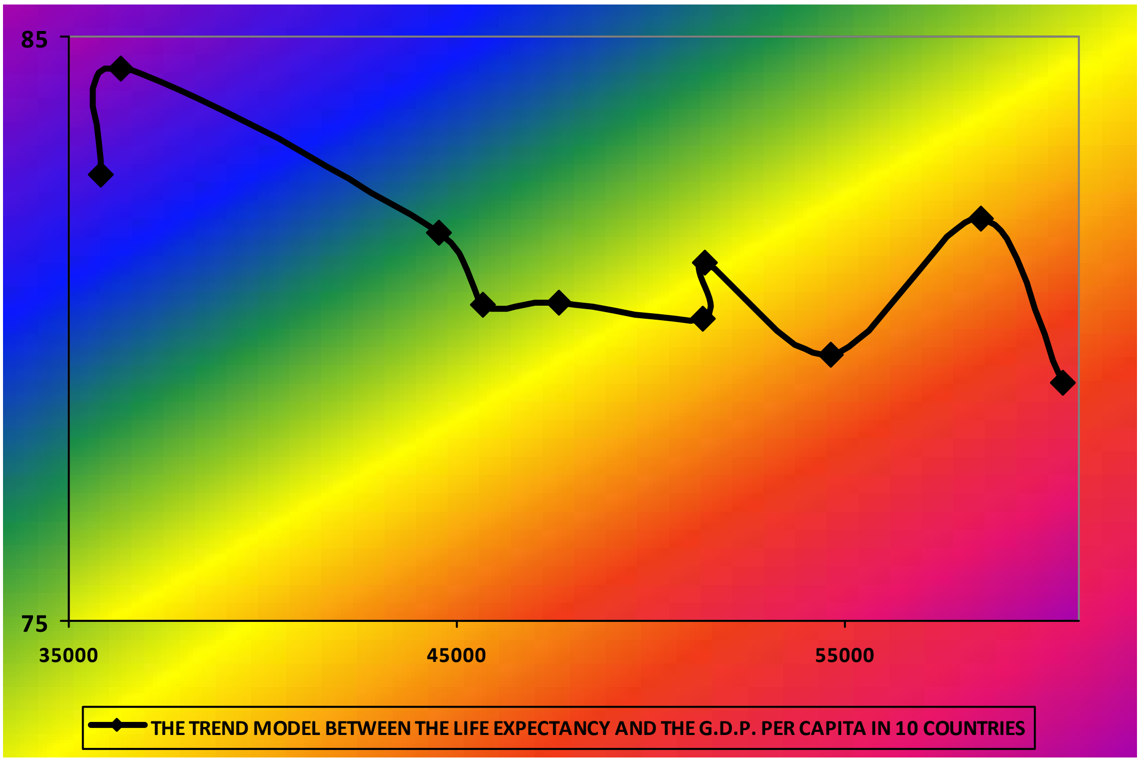

Figure 1. The trend model of the values regarding the correlation between the Life Expectancy and the GDP per capita in 10 contries, in 2014

We observe that, the cloud of points which reflects the values of the Life Expectancy in function of the G.D.P. per capita in 10 countries, in 2014, it carrying around a linear model of trend, according to the type no.1. So, the path reflected by the correlation between the Life Expectancy and the G.D.P. per capita in these 10 countries, in 2014, is a linear trend of the shape $y_{t_i} = a + b \cdot x_i$, with other words it confirms the hypothesis H_0.

3. The intensity of the correlation between the G.D.P. per capita and the Life Expectancy in 2014, for 10 countries

For to reflect the intensity of the linear correlation between the G.D.P. per capita and the Life Expectancy in 2014, for 10 countries, we use the Pearson correlation coefficient noted with *r* [3]:

Table 5. The calculation of the value for the correlation coefficient in the case of the linear correlation between the G.D.P. per capita and the Life Expectancy in 2014, for 10 countries

COUNTRIES	THE G.D.P. PER CAPITA (x_i)	THE LIFE EXPECTANCY (y_i)	x_i^2	y_i^2	$x_i y_i$
ITALY	**35.823**	**82,63**	1283287329	6827,7169	2960054,49
JAPAN	**36.332**	**84,46**	1320014224	7133,4916	3068600,72
FRANCE	**44.538**	**81,66**	1983633444	6668,3556	3636973,08
U. K.	**45.653**	**80,42**	2084196409	6467,3764	3671414,26
GERMANY	**47.590**	**80,44**	2264808100	6470,5936	3828139,60
AUSTRIA	**51.307**	**80,17**	2632408249	6427,2289	4113282,19
NETHERLANDS	**51.373**	**81,12**	2639185129	6580,4544	4167377,76
U.S.A.	**54.597**	**79,56**	2980832409	6329,7936	4343737,32
SWEDEN	**58.491**	**81,89**	3421197081	6705,9721	4789827,99
DENMARK	**60.564**	**79,09**	3667998096	6255,2281	4790006,76
TOTAL	**486268**	**811,44**	24277557470	65866,2112	39369414,17

$$r = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \cdot \sum_{i=1}^{n} y_i}{\sqrt{[n\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2][n\sum_{i=1}^{n} y_i^2 - (\sum_{i=1}^{n} y_i)^2]}} =$$

$$= \frac{10 \cdot 39.369.414{,}17 - 486.268 \cdot 811{,}44}{\sqrt{[10 \cdot 24.277.557.470 - (486.268)^2][10 \cdot 65.866{,}2112 - (811{,}44)^2]}} = -0{,}74$$

In conclusion, because the value of the Pearson correlation coefficient tends to -1, there is a strong intensity of the relationship between the G.D.P. per capita and the Life Expectancy in 2014, for respectively 10 countries.

4. The modeling of the trend between the G.D.P. per capita and the Life Expectancy between 2010-2014, for Romania

In the period 2010-2014, we observe the next evolution concerning the G.D.P. per capita and the Life Expectancy in Romania, according to the table no. 6:

Table 6 The evolution of the G.D.P. per capita and the Life Expectancy between 2010-2014, in Romania

YEARS	GROSS DOMESTIC PRODUCT PER CAPITA IN ROMANIA (Euro)	LIFE EXPECTANCY IN ROMANIA
2010	**6.300**	**73,74**
2011	**6.400**	**73,98**
2012	**6.400**	**74,22**
2013	**6.700**	**74,45**
2014	**6.900**	**74,69**

The sourse: „EUROSTAT ", CIA World Factbook

We want to identify the trend model between the G.D.P. per capita and the Life Expectancy for Romania, in the period 2010-2014, using the table no. 6.

- if we formulate the null hypothesis H_0 *:* which mentions the assumption of the existence for the model of tendency concerning ω factor, where ω *= the Life Expectancy in Romania,* as being the function $\omega_{t_i} = a + b \cdot \xi_i$, then the parameters *a* and *b* of the adjusted linear function, can to be calculated by means of the next system [1]:

$$\begin{cases} n \cdot a + b\sum_{i=1}^{n} \xi_i = \sum_{i=1}^{n} \omega_i \\ a\sum_{i=1}^{n} \xi_i + b \cdot \sum_{i=1}^{n} \xi_i^2 = \sum_{i=1}^{n} \xi_i \cdot \omega_i \end{cases}$$

Therefore,

$$a = \frac{\sum_{i=1}^{n} \xi_i^2 \sum_{i=1}^{n} \omega_i - \sum_{i=1}^{n} \xi_i \sum_{i=1}^{n} \xi_i \omega_i}{n\sum_{i=1}^{n} \xi_i^2 - (\sum_{i=1}^{n} \xi_i^2)}$$

$$b = \frac{n\sum_{i=1}^{n} \xi_i\, \omega_i - \sum_{i=1}^{n} \xi_i \sum_{i=1}^{n} \omega_i}{n\sum_{i=1}^{n} \xi_i^2 - (\sum_{i=1}^{n} \xi_i^2)}$$

Table 7 The estimate of the value for the variation coefficient in the case of the adjusted linear function, in the hypothesis concerning the linear evolution of the correlation between the G.D.P. per capita in Romania and the Life Expectancy in Romania, between 2010-2014

YEARS	G.D.P. per capita in Romania (Euro) (ξ_i)	LIFE EXPECTANCY in Romania (ω_i)	*LINEAR TREND*			
			ξ_i^2	$\xi_i \omega_i$	$\omega_{\xi_i} = a + b\xi_i$	$\lvert\omega_i - \omega_{\xi_i}\rvert$
2010	**6.300**	**73,74**	39.690.000	464.562	73,8780952381	0,14
2011	**6.400**	**73,98**	40.960.000	473.472	74,0188888381	0,04
2012	**6.400**	**74,22**	40.960.000	475.008	74,0188888381	0,20
2013	**6.700**	**74,45**	44.890.000	498.815	74,4412697881	0,01
2014	**6.900**	**74,69**	47.610.000	515.361	74,7228570881	0,03
TOTAL	**32.700**	**371,08**	214.110.000	2.427.218	371,0799997905	0,42

If we calculate the statistical data for to adjust the linear function, we obtain for the parameters *a* and *b* the values:

$$a = \frac{214.110.000 \cdot 371{,}08 - 32.700 \cdot 2.427.218}{5 \cdot 214.110.000 - (32.700)^2} = 65{,}0080952381$$

$$b = \frac{5 \cdot 2.427.218 - 32.700 \cdot 371{,}08}{5 \cdot 214.110.000 - (32.700)^2} = 0{,}0014079365$$

Hence, the coefficient of variation for the adjusted linear function is:

$$v_I = \left[\frac{\sum_{i=-m}^{m} \left| \omega_i - \omega_{\xi_i}^I \right|}{n} : \frac{\sum_{i=-m}^{m} \omega_i}{n} \right] \cdot 100 = \frac{\sum_{i=-m}^{m} \left| \omega_i - \omega_{\xi_i}^I \right|}{\sum_{i=-m}^{m} \omega_i} \cdot 100 = \frac{0{,}42}{371{,}08} \cdot 100 = 0{,}11\%$$

- in the situation of the alternative hypothesis H_1 *:* which specifies the assumption of the existence for the model of tendency regarding ω factor, where ω *= the Life Expectancy in Romania*, as being the quadratic function $\omega_{\xi_i} = a + b \cdot \xi_i + c\xi_i^2$, the parameters *a*, *b* și *c* of the adjusted quadratic function, can to be calculated by means of the system [1]:

$$S = \sum_{i=1}^{n} (\omega_i - \omega_{\xi i})^2 = \min \Leftrightarrow S = \sum_{i=1}^{n} (\omega_i - a - b\xi_i - c\xi_i^2)^2 = \min$$

$$\begin{cases} \frac{\partial S}{\partial a} = 0 \\ \frac{\partial S}{\partial b} = 0 \\ \frac{\partial S}{\partial c} = 0 \end{cases} \Rightarrow \begin{cases} 2\sum_{1=1}^{n} (\omega_i - a - b\xi_i - c\xi_i^2)(-1) = 0 / (-\frac{1}{2}) \\ 2\sum_{1=1}^{n} (\omega_i - a - b\xi_i - c\xi_i^2)(-\xi_i) = 0 / (-\frac{1}{2}) \\ 2\sum (\omega_i - a - b\xi_i - c\xi_i^2)(-\xi_i^2) = 0 / (-\frac{1}{2}) \end{cases} \Rightarrow$$

Therefore,

$$\begin{cases} n \cdot a + b\sum_{i=1}^{n}\xi_i + c\sum_{i=1}^{n}\xi_i^2 = \sum_{i=1}^{n}\omega_i \\ a\sum_{i=1}^{n}\xi_i + b\cdot\sum_{i=1}^{n}\xi_i^2 + c\sum_{i=1}^{n}\xi_i^3 = \sum_{i=1}^{n}\xi_i\cdot\omega_i \\ a\cdot\sum_{i=1}^{n}\xi_i^2 + b\sum_{i=1}^{n}\xi_i^3 + c\sum_{i=1}^{n}\xi_i^4 = \sum_{i=1}^{n}\xi_i^2\cdot\omega_i \end{cases}$$

Table 8. The estimates of the value for the variation coefficient in the case of the adjusted quadratic function, in the hypothesis concerning the parabolic evolution of the correlation between the GDP per capita and the Life Expectancy in 10 countries, in 2014

YEARS	G.D.P. per capita in Romania (Euro) (ξ_i)	LIFE EXPECTANCY in Romania (ω_i)	*PARABOLIC TREND*				
			ξ_i^3	ξ_i^4	$\xi_i^2\omega_i$	$\omega_{\xi_i} = a + b\xi_i + c$	$\lvert\omega_i - \omega_{\xi_i}$
2010	**6.300**	**73,74**	250.047.000.000	1.575.296.100.000.000	2.926.740.600	77,423799464	3,68
2011	**6.400**	**73,98**	262.144.000.000	1.677.721.600.000.000	3.030.220.800	72,746168851	1,23
2012	**6.400**	**74,22**	262.144.000.000	1.677.721.600.000.000	3.030.220.800	72,746168851	1,47
2013	**6.700**	**74,45**	300.763.000.000	2.015.112.100.000.000	3.342.060.500	70,168297024	4,28
2104	**6.900**	**74,69**	328.509.000.000	2.266.712.100.000.000	3.555.990.900	77,995565804	3,31
TOTAL			1.403.607.000.000	9.212.563.500.000.000	15.885.233.600	371,079999994	13,97

If we calculate the statistical data for to adjust the quadratic function, we obtain for the parameters *a, b* and *c* the next values:

$$\begin{cases} 5\cdot a + 32.700\cdot b + 214.110.000\cdot c = 371{,}08 \\ 32.700\cdot a + 214.110.000\cdot b + 1.403.607.000.000\cdot c = 2.427.218 \\ 214.110.000\cdot a + 1.403.607.000.000\cdot b + 9.212.563.500.000.000\cdot c = 15.885.233.600 \end{cases} \Rightarrow$$

$$a = 4.221{,}001247894 \qquad b = -1{,}2590992561 \qquad c = 0{,}0000954585$$

So, the coefficient of variation for the adjusted quadratic function has the value:

$$v_{II} = \left[\frac{\sum_{i=-m}^{m}\left|\omega_i - \omega_{\xi_i}^{II}\right|}{n} : \frac{\sum_{i=-m}^{m}\omega_i}{n}\right]\cdot 100 = \frac{\sum_{i=-m}^{m}\left|\omega_i - \omega_{\xi_i}^{II}\right|}{\sum_{i=-m}^{m}\omega_i}\cdot 100 = \frac{13{,}97}{371{,}08}\cdot 100 = 3{,}77\%$$

- in the case of the alternative hypothesis H_2 *:* which describes the supposition the assumption of the existence for the model of tendency concerning ω factor, where ω *= the Life Expectancy in Romania*, as being the exponential function $\omega_{\xi_i} = ab^{\xi_i}$, then the parameters *a* and *b* of the adjusted exponential function, can to be calculated by means of the next system [1]:

$$S = \sum_{i=1}^{n}(\lg\omega_i - \lg\omega_{\xi_i})^2 = \min \Leftrightarrow S = \sum_{i=1}^{n}(\lg\omega_i - \lg a - \xi_i\lg b)^2 = \min$$

$$\begin{cases} \dfrac{\partial S}{\partial \lg a} = 0 \\ \dfrac{\partial S}{\partial \lg b} = 0 \end{cases} \Rightarrow \begin{cases} 2\sum_{1=1}^{n}(\lg \omega_i - \lg a - \xi_i \lg b)(-1) = 0 /(-\dfrac{1}{2}) \\ 2\sum_{1=1}^{n}(\lg \omega_i - \lg a - \xi_i \lg b)(-\xi_i) = 0 /(-\dfrac{1}{2}) \end{cases} \Rightarrow$$

$$\begin{cases} n \cdot \lg a + \lg b \cdot \sum_{i=1}^{n} \xi_i = \sum_{i=1}^{n} \lg \omega_i \\ \lg a \sum_{i=1}^{n} \xi_i + \lg b \cdot \sum_{i=1}^{n} \xi_i^2 = \sum_{i=1}^{n} \xi_i \cdot \lg \omega_i \end{cases}$$

Thus,

$$\lg a = \frac{\begin{vmatrix} \sum_{i=1}^{n} \lg \omega_i & \sum_{i=1}^{n} \xi_i \\ \sum_{i=1}^{n} \xi_i \lg \omega_i & \sum_{i=1}^{n} \xi_i^2 \end{vmatrix}}{\begin{vmatrix} n & \sum_{i=1}^{n} \xi_i \\ \sum_{i=1}^{n} \xi_i & \sum_{i=1}^{n} \xi_i^2 \end{vmatrix}} = \frac{\sum_{i=1}^{n} \lg \omega_i \sum_{i=1}^{n} \xi_i^2 - \sum_{i=1}^{n} \xi_i \lg \omega_i \sum_{i=1}^{n} \xi_i}{n \sum_{i=1}^{n} \xi_i^2 - \left(\sum_{i=1}^{n} \xi_i \right)^2}$$

and

$$\lg b = \frac{\begin{vmatrix} n & \sum_{i=1}^{n} \lg \omega_i \\ \sum_{i=1}^{n} \xi_i & \sum_{i=1}^{n} \xi_i \lg \omega_i \end{vmatrix}}{\begin{vmatrix} n & \sum_{i=1}^{n} \xi_i \\ \sum_{i=1}^{n} \xi_i & \sum_{i=1}^{n} \xi_i^2 \end{vmatrix}} = \frac{n \cdot \sum_{i=1}^{n} \xi_i \lg \omega_i - \sum_{i=1}^{n} \lg \omega_i \sum_{i=1}^{n} \xi_i}{n \sum_{i=1}^{n} \xi_i^2 - \left(\sum_{i=1}^{n} \xi_i \right)^2}$$

Table 9 The estimate of the value for the variation coefficient in the case of the adjusted exponential function, in the hypothesis concerning the exponential evolution of the correlation between the G.D.P. per capita in Romania and the Life Expectancy in Romania

YEARS	G.D.P. per capita Romania (Euro) (ξ_i)	Life Expectancy (ω_i)	*EXPONENTIAL TREND*				
			$\lg \omega_i$	$\xi_i \lg \omega_i$	$\lg \omega_{\xi i} = \lg a + \xi_i \lg b$	$\omega_{\xi i} = ab^{\xi_i}$	$\lvert \omega_i - \omega_{\xi_i} \rvert$
2010	**6.300**	**73,74**	1,8677031333	11.766,529739601	1,8535167847	71,37017884	2,37
2011	**6.400**	**73,98**	1,8691143270	11.962,331692668	1,8541024487	71,46648922	2,51
2012	**6.400**	**74,22**	1,8705209500	11.971,334080081	1,8541024487	71,4668922	2,75
2013	**6.700**	**74,45**	1,8718647021	12.541,493503990	1,8558585887	71,75606077	2,69
2014	**6.900**	**74,69**	1,8732624594	12.925,510970127	1,8570293487	71,9497599	2,74
TOTAL	**32.700**	**371,08**	9,3524655718	61.167,199986467			13,06

Consequently, if we calculate the statistical data for to adjust the exponential function, we obtain for the parameters *a* and *b* the values:

$$\lg a = \frac{\begin{vmatrix} 9{,}352465571861 & 32.700 \\ 61.167{,}199986467 & 214.110.000 \end{vmatrix}}{\begin{vmatrix} 5 & 32.700 \\ 32.700 & 214.110.000 \end{vmatrix}} = 1{,}8166381287$$

$$\lg b = \frac{\begin{vmatrix} 5 & 9{,}352465571861 \\ 32.700 & 61.167{,}199986467 \end{vmatrix}}{\begin{vmatrix} 5 & 32.700 \\ 32.700 & 214.110.000 \end{vmatrix}} = 0{,}0000058538$$

Accordingly, the coefficient of variation for the adjusted exponential function has the next value:

$$v_{\exp} = \left[\frac{\sum_{i=-m}^{m} \left| \omega_i - \omega_{\xi_i}^{\exp} \right|}{n} : \frac{\sum_{i=-m}^{m} \omega_i}{n} \right] \cdot 100 = \frac{\sum_{i=-m}^{m} \left| \omega_i - \omega_{\xi_i}^{\exp} \right|}{\sum_{i=-m}^{m} \omega_i} \cdot 100 = \frac{13{,}06}{371{,}08} \cdot 100 = 3{,}52\%$$

We apply the coefficients of variation method as criterion of selection for the best model of trend.
We notice that:

$$v_I = 0{,}11\% < v_{\exp} = 3{,}52\% < v_{II} = 3{,}77\%$$

So, the path reflected by the correlation between the Life Expectancy in Romania and the G.D.P. per capita in Romania, between 2010-2014, is a linear trend of the shape $\omega_{\xi_i} = a + b \cdot \xi_i$, with other words it confirms the hypothesis H_0.

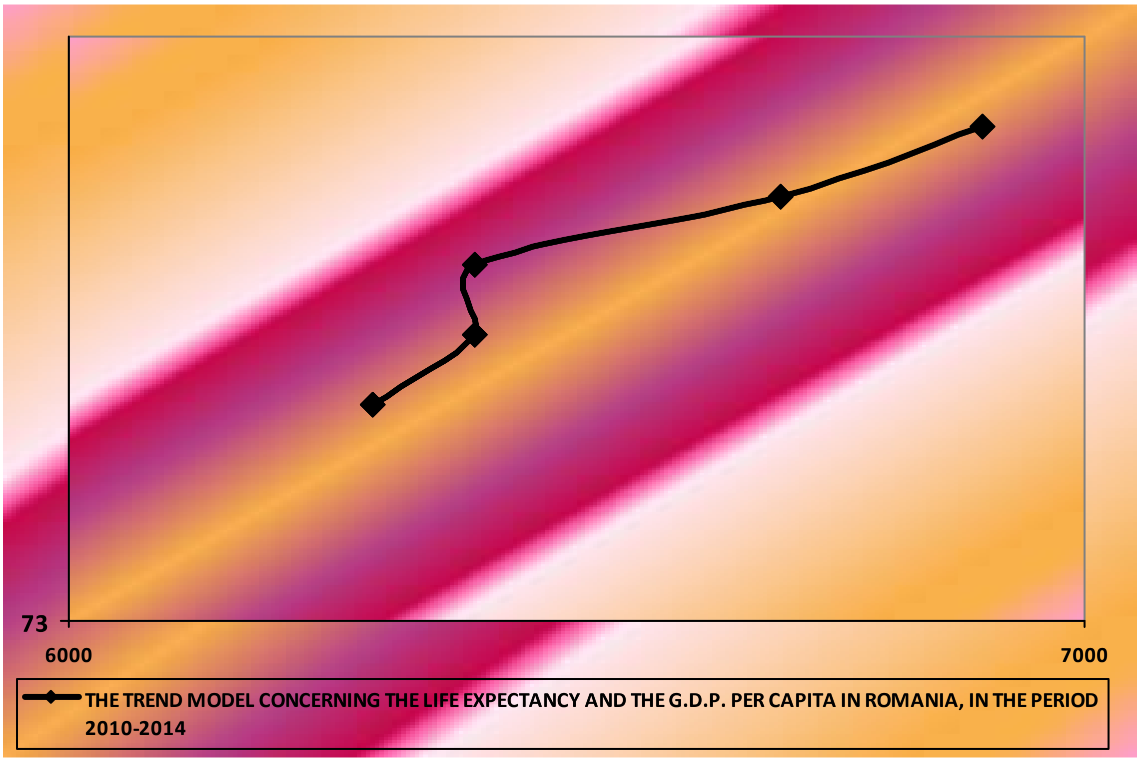

Figure 2. The trend model of the values for the correlation between the Life Expectancy and the G.D.P. per capita in Romania, in the period 2010-2014

We observe that, the cloud of points which reflects the values of the the Life Expectancy in Romania in function of the G.D.P. per capita in Romania, between 2010-2014, it carrying around a linear model of trend, according to the type no.2.

5. The intensity of the correlation between the G.D.P. per capita in Romania and the Life Expectancy in Romania, in the period 2010-2014

For to reflect the intensity of the linear correlation between the G.D.P. per capita in Romania and the Life Expectancy in Romania, between 2010-2014, we use the Pearson correlation coefficient noted with *r* [3]:

Table 10 The calculation of the value for the correlation coefficient in the case of the linear correlation between the G.D.P. per capita in Romania and the Life Expectancy in Romania, between 2010-2014

YEARS	THE G.D.P. PER CAPITA IN ROMANIA (ξ_i)	THE LIFE EXPECTANCY IN ROMANIA (ω_i)	ξ_i^2	ω_i^2	$\xi_i \omega_i$
2010	**6.300**	**73,74**	39.690.000	5.437,5876	464.562
2011	**6.400**	**73,98**	40.960.000	5.473,0404	473.472
2012	**6.400**	**74,22**	40.960.000	5.508,6084	475.008
2013	**6.700**	**74,45**	44.890.000	5.542,8025	498.815
2014	**6.900**	**74,69**	47.610.000	5.578,5961	515.361
TOTAL	**32.700**	**371,08**	214.110.000	27.540,635	2.427.218

$$r = \frac{n\sum_{i=1}^{n}\xi_i\omega_i - \sum_{i=1}^{n}\xi_i \cdot \sum_{i=1}^{n}\omega_i}{\sqrt{[n\sum_{i=1}^{n}\xi_i^2 - (\sum_{i=1}^{n}\xi_i)^2][n\sum_{i=1}^{n}\omega_i^2 - (\sum_{i=1}^{n}\omega_i)^2]}} =$$

$$= \frac{5 \cdot 2.427.218 - 371{,}08 \cdot 32.700}{\sqrt{[5 \cdot 214.110.000 - (32.700)^2][5 \cdot 27.540{,}635 - (371{,}08)^2]}} = 0{,}94$$

In conclusion, because the value of the Paerson correlation coefficient tends to 1, there is a very strong intensity of the relationship between the G.D.P. per capita in Romania and the Life Expectancy in Romania, between 2010-2014.

6. Conclusions

We can to synthesize that, there is a strong intensity of the correlation between the G.D.P. per capita and the Life Expectancy in the respectively analysed 10 countries in 2014 and a very strong intensity of the relationship between the G.D.P. per capita and the Life Expectancy in Romania, in the period 2010-2014, because the G.D.P. per capita represents a measure of the societal progress and the effects of him development can be observed on the Life Expectancy.

References

[1]. Gauss C. F. - „Theoria Combinationis Observationum Erroribus Minimis Obnoxiae", Apud Henricum Dieterich Publising House, Gottingae, 1823.

[2]. Kariya T., Kurata H. - „Generalized Least Squares", John Wiley&Sons Publishing House, Hoboken, 2004.

[3]. Pearson K. - „On the General Theory of skew correlation and non-linear regression", Dulau&Co Publishing House, London, 1905.

[4]. Rao Radhakrishna C., Toutenburg H., Fieger A., Heuman C., Nittner T., Scheid S. - „Linear Models: Least Squares and Alternatives", Springer Series in Statistics, Springer-Verlag New York – Berlin Heidelberg Publishing House, New-York, 1999.

[5]. Wolberg J. - „Data Analysis Using the method of Least Squares: Extracting the Most Information from Experiments", Springer-Verlag New York – Berlin Heidelberg Publishing House, New-York, 2006.