Annals of "Dunarea de Jos" University of Galati
Fascicle I. Economics and Applied Informatics
Years XXI – nº1/2015
ISSN-L 1584-0409 ISSN-Online 2344-441X
www.eia.feaa.ugal.ro

Statistical Approaches Regarding the Evolution of the Earthquakes in Romania

Gabriela OPAIT *

ARTICLE INFO

Article history:
Accepted April 2015
Available online May 2015
JEL Classification
C1, C12, C2

Keywords:
Least Squares Method, Earthquake, Magnitude, Depth, Forecasting

ABSTRACT

This paper reflects the statistical modeling of the values regarding the magnitudes of the earthquakes in Romania, respectively the depth of the earthquakes, through by means of the „Least Squares Method", which is a method through we can to reflect the trend line of the best fit concerning a model. If we achieve in time and space an analysis concerning the earthquakes, we can to say that any earthquake has an unexpected character and the fortuitous factor playes the principal role.

1. Introduction

This research reflects a statistical analysis of the trends concerning the magnitude of the earthquakes in Romania, respectively the depth of the earthquakes between the years 1940-2014. The purpose of the research reflects the possibility for to anticipate the values concerning the evolution in future of the magnitude for these earthquakes, respectively, the depth of the earthquakes by means of the forecasting methods. The statistical methods used are the „Coefficients of Variation Method", the „Least Squares Method" applied for to calculate the parameters of the regression equation and the Forecasting Method through the „Least Squares Method". The sections 2, respectively the section 3, present the methodology for to achieve the trends models for the magnitude of the earthquakes, respectively for the depth of the earthquakes, with the help of the „Least Squares Method". The section 4 expresses the forecasting method through the „Least Squares Method". The state of the art in this domain is represented by the research belongs to Carl Friederich Gauss, who created the „Least Squares Method" [1].

2. The modeling of the trend for the evolution regarding the magnitude of the earthquakes in Romania, between 1940-2014

In the period 1940-2014, we observe the next evolution concerning the magnitude of the earthquakes in Romania with the epicentre in Vrancea, according to the table no. 1:

Table no. 1 The evolution of the magnitudes concerning the earthquakes in Romania, between 1940-2014

YEARS	THE MAGNITUDE OF THE EARTHQUAKES (RICHTER)
1940	7,7
1977	7,4
1986	7,1
1990	6,9
2004	6,0
2014	5,6

Source: „National Institute of Seismology from Romania"

We want to identify the trend model for the magnitudes of the earthquakes in Romania, in the period 1940-2014, using the table no.1.

- if we formulate the null hypothesis H_0 *:* which mentions the assumption of the existence for the model of tendency of the factor *X = the magnitude of the earthquakes in Romania,* as being the function $x_{t_i} = a + b \cdot t_i$, then the parameters *a* and *b* of the adjusted linear function, can to be calculated by means of the next system [1]:

* Faculty of Economics and Business Administration, "Dunarea de Jos" University of Galati, Romania. E-mail address: Gabriela.opait@ugal.ro (G. Opait)

$$S=\sum_{i=1}^{n}(x_i-x_{ti})^2=\min \Leftrightarrow S=\sum_{i=1}^{n}(x_i-a-bt_i)^2=\min$$

$$\begin{cases}\dfrac{\partial S}{\partial a}=0\\ \dfrac{\partial S}{\partial b}=0\end{cases}\Rightarrow\begin{cases}2\sum_{1=1}^{n}(x_i-a-bt_i)(-1)=0/(-\dfrac{1}{2})\\ 2\sum_{1=1}^{n}(x_i-a-bt_i)(-t_i)=0/(-\dfrac{1}{2})\end{cases}\Rightarrow\begin{cases}na+b\sum_{i=1}^{n}t_i=\sum_{i=1}^{n}x_i\\ a\sum_{i=1}^{n}t_i+b\sum_{i=1}^{n}t_i^2=\sum_{i=1}^{n}x_it_i\end{cases}\Rightarrow$$

Therefore,

$$a=\frac{\begin{vmatrix}\sum_{i=1}^{n}x_i & \sum_{i=1}^{n}t_i\\ \sum_{i=1}^{n}x_it_i & \sum_{i=1}^{n}t_i^{2}\end{vmatrix}}{\begin{vmatrix}n & \sum_{i=1}^{n}t_i\\ \sum_{i=1}^{n}t_i & \sum_{i=1}^{n}t_i^{2}\end{vmatrix}}=\frac{\sum_{i=1}^{n}x_i\sum_{i=1}^{n}t_i^2-\sum_{i=1}^{n}x_it_i\sum_{i=1}^{n}t_i}{n\sum_{i=1}^{n}t_i^2-\left(\sum_{i=1}^{n}t_i\right)^2}\quad b=\frac{\begin{vmatrix}n & \sum_{i=1}^{n}x_i\\ \sum_{i=1}^{n}t_i & \sum_{i=1}^{n}x_it_i\end{vmatrix}}{\begin{vmatrix}n & \sum_{i=1}^{n}t_i\\ \sum_{i=1}^{n}t_i & \sum_{i=1}^{n}t_i^{2}\end{vmatrix}}=\frac{n\sum_{i=1}^{n}x_it_i-\sum_{i=1}^{n}t_i\sum_{i=1}^{n}x_i}{n\sum_{i=1}^{n}t_i^2-\left(\sum_{i=1}^{n}t_i\right)^2}$$

Table no. 2 The estimate of the value for the variation coefficient in the case of the adjusted linear function, in the hypothesis concerning the linear evolution of the magnitude of the earthquakes in Romania, between the years 1940-2014

YEARS	THE MAGNITUDE OF THE EARTHQUAKES IN ROMANIA (RICHTER) (x_i)	*LINEAR TREND*				
		t_i	t_i^2	t_ix_i	$x_{t_i}=a+bt_i$	$\lvert x_i-x_{t_i}\rvert$
1940	**7,7**	-37	1369	-284,9	8,069951259	0,37
1977	**7,4**	0	0	0	7,015969047	0,38
1986	**7,1**	9	81	63,9	6,759594995	0,34
1990	**6,9**	13	169	89,7	6,645650973	0,25
2004	**6,0**	27	729	162,0	6,246846892	0,25
2014	**5,6**	37	1369	207,2	5,961986835	0,36
TOTAL	**40,7**	49	3717	237,9	40,700000000	1,95

If we calculate the statistical data for to adjust the linear function, we obtain for the parameters *a* and *b* the values:

$$a=\frac{\begin{vmatrix}40{,}7 & 49\\ 237{,}9 & 3717\end{vmatrix}}{\begin{vmatrix}6 & 49\\ 49 & 3717\end{vmatrix}}=\frac{151281{,}9-11657{,}1}{22302-2401}=7{,}015969047$$

$$b=\frac{\begin{vmatrix}6 & 40{,}7\\ 49 & 237{,}9\end{vmatrix}}{\begin{vmatrix}6 & 49\\ 49 & 3717\end{vmatrix}}=\frac{1427{,}4-1994{,}3}{22302-2401}=-0{,}028486005$$

Hence, the coefficient of variation for the adjusted linear function is:

$$v_I=\left[\frac{\sum_{i=-m}^{m}\left|x_i-x_{t_i}^{I}\right|}{n}:\frac{\sum_{i=-m}^{m}x_i}{n}\right]\cdot 100=\frac{\sum_{i=-m}^{m}\left|x_i-x_{t_i}^{I}\right|}{\sum_{i=-m}^{m}x_i}\cdot 100=\frac{1{,}95}{40{,}7}\cdot 100=4{,}79\%$$

- in the situation of the alternative hypothesis H_1 *:* which specifies the assumption of the existence for the model of tendency of the factor *X = the magnitude of the earthquakes in Romania*, as being the quadratic function $x_{t_i} = a + b \cdot t_i + ct_i^2$, the parameters *a*, *b* şi *c* of the adjusted quadratic function, can to be calculated by means of the system [1]:

$$S = \sum_{i=1}^{n} (x_i - x_{ti})^2 = \min \Leftrightarrow S = \sum_{i=1}^{n} (x_i - a - bt_i - ct_i^2)^2 = \min$$

$$\begin{cases} \dfrac{\partial S}{\partial a} = 0 \\ \dfrac{\partial S}{\partial b} = 0 \Rightarrow \\ \dfrac{\partial S}{\partial c} = 0 \end{cases} \begin{cases} 2\sum_{1=1}^{n} (x_i - a - bt_i - ct_i^2)(-1) = 0 / (-\dfrac{1}{2}) \\ 2\sum_{1=1}^{n} (x_i - a - bt_i - ct_i^2)(-t_i) = 0 / (-\dfrac{1}{2}) \Rightarrow \\ 2\sum (x_i - a - bt_i - ct_i^2)(-t_i^2) = 0 / (-\dfrac{1}{2}) \end{cases}$$

Therefore,

$$\begin{cases} n \cdot a + b\sum_{i=1}^{n} t_i + c\sum_{i=1}^{n} t_i^2 = \sum_{i=1}^{n} x_i \\ a\sum_{i=1}^{n} t_i + b \cdot \sum_{i=1}^{n} t_i^2 + c\sum_{i=1}^{n} t_i^3 = \sum_{i=1}^{n} t_i \cdot x_i \\ a \cdot \sum_{i=1}^{n} t_i^2 + b\sum_{i=1}^{n} t_i^3 + c\sum_{i=1}^{n} t_i^4 = \sum_{i=1}^{n} t_i^2 \cdot x_i \end{cases}$$

Table no. 3 The estimates of the value for the variation coefficient in the case of the adjusted quadratic function, in the hypothesis concerning the parabolic evolution of the magnitude of the earthquakes in Romania, between the years 1940-2014

YEARS	THE MAGNITUDE OF THE EARTHQUAKES IN ROMANIA (RICHTER) (x_i)	*PARABOLIC TREND*						
		t_i	t_i^2	t_i^3	t_i^4	$t_i^2 x_i$	$x_{t_i} = a + bt_i + ct_i^2$	$\lvert x_i - x_{t_i} \rvert$
1940	**7,7**	-37	1369	-50653	1874161	10541,3	7,711011419	0,01
1977	**7,4**	0	0	0	0	0	7,371877263	0,03
1986	**7,1**	9	81	729	6561	575,1	7,058642685	0,04
1990	**6,9**	13	169	2197	28561	1166,1	6,890445169	0,01
2004	**6,0**	27	729	19683	531441	4374	6,161301915	0,16
2014	**5,6**	37	1369	50653	1874161	7666,4	5,506721545	0,09
TOTAL	**40,7**	49	3717	22609	4314885	24322,9	40,700000000	0,34

If we calculate the statistical data for to adjust the quadratic function, we obtain for the parameters *a, b* and *c* the next values:

$$\begin{cases} 6 \cdot a + 49 \cdot b + 3717 \cdot c = 40{,}7 \\ 49 \cdot a + 3717 \cdot b + 22609 \cdot c = 237{,}9 \\ 3717 \cdot a + 22609 \cdot b + 4314885 \cdot c = 24322{,}9 \end{cases} \Rightarrow$$

$a = 7{,}371877263$ $\quad b = -0{,}029787701$ $\quad c = -0{,}000557349$

So, the coefficient of variation for the adjusted quadratic function has the value:

$$v_{II} = \left[\frac{\sum_{i=-m}^{m} \left| x_i - x_{t_i}^{II} \right|}{n} : \frac{\sum_{i=-m}^{m} x_i}{n} \right] \cdot 100 = \frac{\sum_{i=-m}^{m} \left| x_i - x_{t_i}^{II} \right|}{\sum_{i=-m}^{m} x_i} \cdot 100 = \frac{0{,}34}{40{,}7} \cdot 100 = 0{,}84\%$$

- in the case of the alternative hypothesis H_2 *:* which describes the supposition the assumption of the existence for the model of tendency of the factor *X = the magnitude of the earthquakes in Romania*, as being the exponential function $x_{t_i} = ab^{t_i}$, then the parameters *a* and *b* of the adjusted exponential function, can to be calculated by means of the next system [1]:

$$S = \sum_{i=1}^{n} (\lg x_i - \lg x_{t_i})^2 = \min \Leftrightarrow S = \sum_{i=1}^{n} (\lg x_i - \lg a - t_i \lg b)^2 = \min$$

$$\begin{cases} \dfrac{\partial S}{\partial \lg a} = 0 \\ \dfrac{\partial S}{\partial \lg b} = 0 \end{cases} \Rightarrow \begin{cases} 2\sum_{1=1}^{n} (\lg x_i - \lg a - t_i \lg b)(-1) = 0/(-\frac{1}{2}) \\ 2\sum_{1=1}^{n} (\lg x_i - \lg a - t_i \lg b)(-t_i) = 0/(-\frac{1}{2}) \end{cases} \Rightarrow$$

$$\begin{cases} n \cdot \lg a + \lg b \cdot \sum_{i=1}^{n} t_i = \sum_{i=1}^{n} \lg x_i \\ \lg a \sum_{i=1}^{n} t_i + \lg b \cdot \sum_{i=1}^{n} t_i^2 = \sum_{i=1}^{n} t_i \cdot \lg x_i \end{cases}$$

Thus,

$$\lg a = \frac{\begin{vmatrix} \sum_{i=1}^{n} \lg x_i & \sum_{i=1}^{n} t_i \\ \sum_{i=1}^{n} t_i \lg x_i & \sum_{i=1}^{n} t_i^{\,2} \end{vmatrix}}{\begin{vmatrix} n & \sum_{i=1}^{n} t_i \\ \sum_{i=1}^{n} t_i & \sum_{i=1}^{n} t_i^{\,2} \end{vmatrix}} = \frac{\sum_{i=1}^{n} \lg x_i \sum_{i=1}^{n} t_i^{\,2} - \sum_{i=1}^{n} t_i \lg x_i \sum_{i=1}^{n} t_i}{n\sum_{i=1}^{n} t_i^{\,2} - \left(\sum_{i=1}^{n} t_i\right)^2}$$

and

$$\lg b = \frac{\begin{vmatrix} n & \sum_{i=1}^{n} \lg x_i \\ \sum_{i=1}^{n} t_i & \sum_{i=1}^{n} t_i \lg x_i \end{vmatrix}}{\begin{vmatrix} n & \sum_{i=1}^{n} t_i \\ \sum_{i=1}^{n} t_i & \sum_{i=1}^{n} t_i^{\,2} \end{vmatrix}} = \frac{n \cdot \sum_{i=1}^{n} t_i \lg x_i - \sum_{i=1}^{n} \lg x_i \sum_{i=1}^{n} t_i}{n\sum_{i=1}^{n} t_i^{\,2} - \left(\sum_{i=1}^{n} t_i\right)^2}$$

Table no. 4 The estimate of the value for the variation coefficient in the case of the adjusted exponential function, in the hypothesis concerning the exponential evolution of the earthquakes in Romania, between 1940-2014

YEARS	THE MAGNITUDE OF THE EARTH-QUAKES IN ROMANIA (RICHTER) (x_i)	EXPONENTIAL TREND					
		t_i	$\lg x_i$	$t_i \lg x_i$	$\lg x_{ti} = \lg a + t_i \lg b$	$x_{ti} = ab^{t_i}$	$\lvert x_i - x_{t_i} \rvert$
1940	**7,7**	-37	0,886490725	-32,80015683	0,912398319	8,173316544	0,47
1977	**7,4**	0	0,869231719	0	0,843829438	6,979582386	0,42
1986	**7,1**	9	0,851258348	7,661325138	0,827150521	6,71661602	0,38
1990	**6,9**	13	0,83884909	10,90503818	0,819737669	6,602944836	0,30
2004	**6,0**	27	0,778151250	21,01008376	0,793792687	6,220032974	0,22
2014	**5,6**	37	0,748188027	27,68295700	0,775260557	5,96019621	0,36
TOTAL	**40,7**	49	4,972169159	34,45924725		40,65	2,15

Consequently, if we calculate the statistical data for to adjust the exponential function, we obtain for the parameters *a* and *b* the values:

$$\lg a = \frac{\begin{vmatrix} 4{,}972169159 & 49 \\ 34{,}45924725 & 3717 \end{vmatrix}}{\begin{vmatrix} 6 & 49 \\ 49 & 3717 \end{vmatrix}} = \frac{18481{,}55276 - 1688{,}503115}{22302 - 2401} = 0{,}843829438$$

$$\lg b = \frac{\begin{vmatrix} 6 & 4{,}972169159 \\ 49 & 34{,}45924725 \end{vmatrix}}{\begin{vmatrix} 6 & 49 \\ 49 & 3717 \end{vmatrix}} = \frac{206{,}7554835 - 243{,}6362888}{22302 - 2401} = -0{,}001853213$$

Accordingly, the coefficient of variation for the adjusted exponential function has the next value:

$$v_{\exp} = \left[\frac{\sum_{i=-m}^{m} \left| x_i - x_{t_i}^{\exp} \right|}{n} : \frac{\sum_{i=-m}^{m} x_i}{n} \right] \cdot 100 = \frac{\sum_{i=-m}^{m} \left| x_i - x_{t_i}^{\exp} \right|}{\sum_{i=-m}^{m} x_i} \cdot 100 = \frac{2{,}15}{40{,}7} \cdot 100 = 5{,}28\%$$

We apply the coefficients of variation method as criterion of selection for the best model of trend.
We notice that:

$$v_{II} = 0{,}84\% < v_I = 4{,}79\% < v_{\exp} = 5{,}28\%$$

So, the path reflected by X factor, which represents the magnitude of the earthquakes in Romania, between 1940-2014, is a parabolic trend of the shape $x_{t_i} = a + b \cdot t_i + ct_i^2$, with other words it confirms the hypothesis H_1.

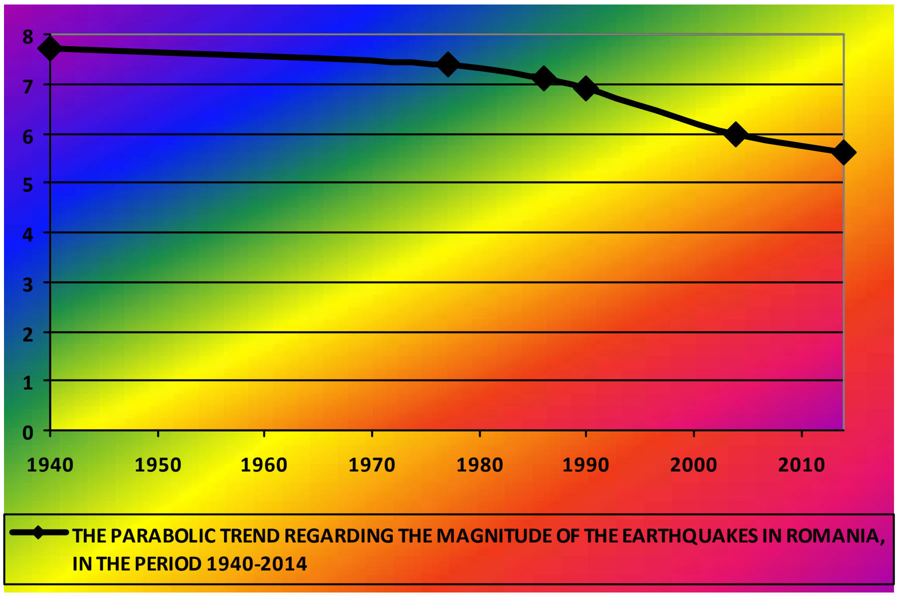

Figure 1. The trend model of the values for the magnitudes of the earthquakes in Romania, in the period 1940-2014

We observe that, the cloud of points which reflects the values of the magnitudes concerning the earthquakes in Romania, between 1940-2014, it carrying around a parabolic model of trend, according to the type no.1.

3. The modeling of the trend for the evolution concerning the depth of the earthquakes in Romania, between 1940-2014

Also, we want to identify the trend model for the depth of the earthquakes in Romania, in the period 1940-2014, using the table no.1.

- if we formulate the null hypothesis H_0 *: which mentions the assumption of the existence for the model of* tendency of the factor *Y = the depth of the earthquakes in Romania,* as being the function $y_{t_i} = a + b \cdot t_i$, then the parameters *a* and *b* of the adjusted linear function, can to be calculated by means of the next system [1]:

$$S = \sum_{i=1}^{n} (y_i - y_{ti})^2 = \min \Leftrightarrow S = \sum_{i=1}^{n} (y_i - a - bt_i)^2 = \min$$

$$\begin{cases} \dfrac{\partial S}{\partial a} = 0 \\ \dfrac{\partial S}{\partial b} = 0 \end{cases} \Rightarrow \begin{cases} 2\sum_{1=1}^{n} (y_i - a - bt_i)(-1) = 0 /(-\dfrac{1}{2}) \\ 2\sum_{1=1}^{n} (y_i - a - bt_i)(-t_i) = 0 /(-\dfrac{1}{2}) \end{cases} \Rightarrow \begin{cases} na + b\sum_{i=1}^{n} t_i = \sum_{i=1}^{n} y_i \\ a\sum_{i=1}^{n} t_i + b\sum_{i=1}^{n} t_i^2 = \sum_{i=1}^{n} y_i t_i \end{cases} \Rightarrow$$

Therefore,

$$a = \frac{\begin{vmatrix} \sum_{i=1}^{n} y_i & \sum_{i=1}^{n} t_i \\ \sum_{i=1}^{n} y_i t_i & \sum_{i=1}^{n} t_i^{\,2} \end{vmatrix}}{\begin{vmatrix} n & \sum_{i=1}^{n} t_i \\ \sum_{i=1}^{n} t_i & \sum_{i=1}^{n} t_i^{\,2} \end{vmatrix}} = \frac{\sum_{i=1}^{n} y_i \sum_{i=1}^{n} t_i^{\,2} - \sum_{i=1}^{n} y_i t_i \sum_{i=1}^{n} t_i}{n\sum_{i=1}^{n} t_i^{\,2} - \left(\sum_{i=1}^{n} t_i\right)^2} \quad b = \frac{\begin{vmatrix} n & \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} t_i & \sum_{i=1}^{n} x_i t_i \end{vmatrix}}{\begin{vmatrix} n & \sum_{i=1}^{n} t_i \\ \sum_{i=1}^{n} t_i & \sum_{i=1}^{n} t_i^{\,2} \end{vmatrix}} = \frac{n\sum_{i=1}^{n} y_i t_i - \sum_{i=1}^{n} t_i \sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} t_i^{\,2} - \left(\sum_{i=1}^{n} t_i\right)^2}$$

Table no. 5 The estimate of the value for the variation coefficient in the case of the adjusted linear function, in the hypothesis concerning the linear evolution for the depth of the earthquakes in Romania, between the years 1940-2014

YEARS	THE DEPTH OF THE EARTHQUAKES IN ROMANIA (KM)	*A. B.* LINEAR TREND				
	(y_i)	t_i	t_i^2	$t_i y_i$	$y_{t_i} = a + bt_i$	$\lvert y_i - y_{t_i} \rvert$
1940	**150,0**	-37	1369	-5550,0	155,2877996	5,59
1977	**94,0**	0	0	0	110,5291593	16,53
1986	**131,4**	9	81	1182,6	99,64192248	31,76
1990	**90,9**	13	169	1181,7	94,80315056	3,90
2004	**98,6**	27	729	2662,2	77,86744884	20,73
2014	**39,0**	37	1369	1443,0	65,77051904	26,77
TOTAL	**603,9**	49	3717	919,5	603,9000000	105,28

If we calculate the statistical data for to adjust the linear function, we obtain for the parameters *a* and *b* the values:

$$a = \frac{\begin{vmatrix} 603{,}9 & 49 \\ 919{,}5 & 3717 \end{vmatrix}}{\begin{vmatrix} 6 & 49 \\ 49 & 3717 \end{vmatrix}} = \frac{2244696{,}3 - 45055{,}5}{22302 - 2401} = 110{,}5291593$$

$$b = \frac{\begin{vmatrix} 6 & 603{,}9 \\ 49 & 919{,}5 \end{vmatrix}}{\begin{vmatrix} 6 & 49 \\ 49 & 3717 \end{vmatrix}} = \frac{5517 - 29591{,}1}{22302 - 2401} = -1{,}20969298$$

Hence, the coefficient of variation for the adjusted linear function is:

$$v_I = \left[\frac{\sum_{i=1}^{n} \left| y_i - y_{t_i}^I \right|}{n} : \frac{\sum_{i=1}^{n} y_i}{n} \right] \cdot 100 = \frac{\sum_{i=1}^{n} \left| y_i - y_{t_i}^I \right|}{\sum_{i=1}^{n} y_i} \cdot 100 = \frac{105,28}{603,9} \cdot 100 = 17,43\%$$

- in the situation of the alternative hypothesis H_1 *: which specifies the assumption of the existence for the model of tendency of the factor Y = the depth of the earthquakes in Romania*, as being the quadratic function $y_{t_i} = a + b \cdot t_i + ct_i^2$, the parameters *a*, *b* şi *c* of the adjusted quadratic function, can to be calculated by means of the system [1]:

$$S = \sum_{i=1}^{n} (y_i - y_{ti})^2 = \min \Leftrightarrow S = \sum_{i=1}^{n} (y_i - a - bt_i - ct_i^2)^2 = \min$$

$$\begin{cases} \dfrac{\partial S}{\partial a} = 0 \\ \dfrac{\partial S}{\partial b} = 0 \\ \dfrac{\partial S}{\partial c} = 0 \end{cases} \Rightarrow \begin{cases} 2\sum_{1=1}^{n} (y_i - a - bt_i - ct_i^2)(-1) = 0 /(-\frac{1}{2}) \\ 2\sum_{1=1}^{n} (y_i - a - bt_i - ct_i^2)(-t_i) = 0 /(-\frac{1}{2}) \\ 2\sum (y_i - a - bt_i - ct_i^2)(-t_i^2) = 0 /(-\frac{1}{2}) \end{cases} \Rightarrow$$

Therefore,

$$\begin{cases} n \cdot a + b\sum_{i=1}^{n} t_i + c\sum_{i=1}^{n} t_i^2 = \sum_{i=1}^{n} y_i \\ a\sum_{i=1}^{n} t_i + b \cdot \sum_{i=1}^{n} t_i^2 + c\sum_{i=1}^{n} t_i^3 = \sum_{i=1}^{n} t_i \cdot y_i \\ a \cdot \sum_{i=1}^{n} t_i^2 + b\sum_{i=1}^{n} t_i^3 + c\sum_{i=1}^{n} t_i^4 = \sum_{i=1}^{n} t_i^2 \cdot y_i \end{cases}$$

Table no. 6 The estimates of the value for the variation coefficient in the case of the adjusted quadratic function, in the hypothesis concerning the parabolic evolution for the depth of the earthquakes in Romania, between the years 1940-2014

YEARS	THE DEPTH OF THE EARTHQUAKES IN ROMANIA (KM) (y_i)	*PARABOLIC TREND*						
		t_i	t_i^2	t_i^3	t_i^4	$t_i^2 y_i$	$y_{t_i} = a + bt_i + ct_i^2$	$\lvert y_i - y_{t_i} \rvert$
1940	**150,0**	-37	1369	-50653	1874161	205350	156,6128331	3,39
1977	**94,0**	0	0	0	0	0	119,1308559	25,13
1986	**131,4**	9	81	729	6561	10643,4	106,8693970	24,53
1990	**90,9**	13	169	2197	28561	15362,1	100,7194104	9,82
2004	**98,6**	27	729	19683	531441	71879,4	75,79997254	22,80
2014	**39,0**	37	1369	50653	1874161	53391,0	54,76753136	15,77
TOTAL	**603,9**	49	3717	22609	4314885	356625,9	603,90000000	101,44

If we calculate the statistical data for to adjust the quadratic function, we obtain for the parameters *a, b* and *c* the next values:

$$\begin{cases} 6 \cdot a + 49 \cdot b + 3717 \cdot c = 603,9 \\ 49 \cdot a + 3717 \cdot b + 22609 \cdot c = 919,5 \\ 3717 \cdot a + 22609 \cdot b + 4314885 \cdot c = 356625,9 \end{cases} \Rightarrow$$

$a = 119,1308559$ $\quad b = -1,241152726$ $\quad c = -0,013470178$

So, the coefficient of variation for the adjusted quadratic function has the value:

$$v_{II} = \left[\frac{\sum_{i=1}^{n} \left| y_i - y_{t_i}^{II} \right|}{n} : \frac{\sum_{i=1}^{n} y_i}{n} \right] \cdot 100 = \frac{\sum_{i=1}^{n} \left| y_i - y_{t_i}^{II} \right|}{\sum_{i=1}^{n} y_i} \cdot 100 = \frac{101{,}44}{603{,}9} \cdot 100 = 16{,}80\%$$

- in the case of the alternative hypothesis H_2 *:* which describes the supposition the assumption of the existence for the model of tendency of the factor *Y= the depth of the earthquakes in Romania*, as being the exponential function $y_{t_i} = ab^{t_i}$, then the parameters *a* and *b* of the adjusted exponential function, can to be calculated by means of the next system [1]:

$$S = \sum_{i=1}^{n} (\lg y_i - \lg y_{t_i})^2 = \min \Leftrightarrow S = \sum_{i=1}^{n} (\lg y_i - \lg a - t_i \lg b)^2 = \min$$

$$\begin{cases} \dfrac{\partial S}{\partial \lg a} = 0 \\ \dfrac{\partial S}{\partial \lg b} = 0 \end{cases} \Rightarrow \begin{cases} 2\sum_{1=1}^{n} (\lg y_i - \lg a - t_i \lg b)(-1) = 0 / (-\frac{1}{2}) \\ 2\sum_{1=1}^{n} (\lg y_i - \lg a - t_i \lg b)(-t_i) = 0 / (-\frac{1}{2}) \end{cases} \Rightarrow$$

$$\begin{cases} n \cdot \lg a + \lg b \cdot \sum_{i=1}^{n} t_i = \sum_{i=1}^{n} \lg y_i \\ \lg a \sum_{i=1}^{n} t_i + \lg b \cdot \sum_{i=1}^{n} t_i^2 = \sum_{i=1}^{n} t_i \cdot \lg y_i \end{cases}$$

Thus,

$$\lg a = \frac{\begin{vmatrix} \sum_{i=1}^{n} \lg y_i & \sum_{i=1}^{n} t_i \\ \sum_{i=1}^{n} t_i \lg y_i & \sum_{i=1}^{n} t_i^2 \end{vmatrix}}{\begin{vmatrix} n & \sum_{i=1}^{n} t_i \\ \sum_{i=1}^{n} t_i & \sum_{i=1}^{n} t_i^2 \end{vmatrix}} = \frac{\sum_{i=1}^{n} \lg y_i \sum_{i=1}^{n} t_i^2 - \sum_{i=1}^{n} t_i \lg y_i \sum_{i=1}^{n} t_i}{n \sum_{i=1}^{n} t_i^2 - \left(\sum_{i=1}^{n} t_i \right)^2}$$

and

$$\lg b = \frac{\begin{vmatrix} n & \sum_{i=1}^{n} \lg y_i \\ \sum_{i=1}^{n} t_i & \sum_{i=1}^{n} t_i \lg y_i \end{vmatrix}}{\begin{vmatrix} n & \sum_{i=1}^{n} t_i \\ \sum_{i=1}^{n} t_i & \sum_{i=1}^{n} t_i^2 \end{vmatrix}} = \frac{n \cdot \sum_{i=1}^{n} t_i \lg y_i - \sum_{i=1}^{n} \lg y_i \sum_{i=1}^{n} t_i}{n \sum_{i=1}^{n} t_i^2 - \left(\sum_{i=1}^{n} t_i \right)^2}$$

Table no. 7 The estimate of the value for the variation coefficient in the case of the adjusted exponential function, in the hypothesis concerning the exponential evolution for the depth of the earthquakes in Romania, between 1940-2014

YEARS	THE DEPTH OF THE EARTH-QUAKES IN ROMANIA (KM) (y_i)	*EXPONENTIAL TREND*					
		t_i	$\lg y_i$	$t_i \lg y_i$	$\lg y_{ti} = \lg a + t_i \lg b$	$y_{ti} = ab^{t_i}$	$\lvert y_i - y_{t_i} \rvert$
1940	**150,0**	-37	2,176091259	-80,51537659	2,237384503	172,7366541	22,74
1977	**94,0**	0	1,973127854	0	2,017161169	104,0306157	10,03
1986	**131,4**	9	2,118595365	19,06735829	1,963593331	91,95880754	39,44
1990	**90,9**	13	1,958563883	25,46133048	1,939785403	87,05333289	3,85
2004	**98,6**	27	1,993876915	53,83467670	1,856457655	71,85510941	26,74
2014	**39,0**	37	1,591064607	58,86939046	1,796937835	62,65241775	23,65
TOTAL	**603,9**	49	11,81131988	76,71737934			126,45

Consequently, if we calculate the statistical data for to adjust the exponential function, we obtain for the parameters *a* and *b* the values:

$$\lg a = \frac{\begin{vmatrix} 11{,}81131988 & 49 \\ 76{,}71737934 & 3717 \end{vmatrix}}{\begin{vmatrix} 6 & 49 \\ 49 & 3717 \end{vmatrix}} = \frac{43902{,}67601 - 3759{,}151588}{22302 - 2401} = 2{,}017161169$$

$$\lg b = \frac{\begin{vmatrix} 6 & 11{,}81131988 \\ 49 & 76{,}71737934 \end{vmatrix}}{\begin{vmatrix} 6 & 49 \\ 49 & 3717 \end{vmatrix}} = \frac{460{,}304276 - 578{,}7546741}{22302 - 2401} = -0{,}005951982$$

Accordingly, the coefficient of variation for the adjusted exponential function has the next value:

$$v_{\exp} = \left[\frac{\sum_{i=1}^{n} \left| y_i - y_{t_i}^{\exp} \right|}{n} : \frac{\sum_{i=1}^{n} y_i}{n} \right] \cdot 100 = \frac{\sum_{i=1}^{n} \left| y_i - y_{t_i}^{\exp} \right|}{\sum_{i=1}^{n} y_i} \cdot 100 = \frac{126{,}45}{603{,}9} \cdot 100 = 20{,}94\%$$

We apply the coefficients of variation method as criterion of selection for the best model of trend.
We notice that:

$$v_{II} = 16{,}80\% < v_I = 17{,}43\% < v_{\exp} = 20{,}94\%$$

So, the path reflected by Y factor, which represents the depth of the earthquakes in Romania, between 1940-2014, is a parabolic trend of the shape $y_{t_i} = a + b \cdot t_i + ct_i^2$, with other words it confirms the hypothesis H_1 .

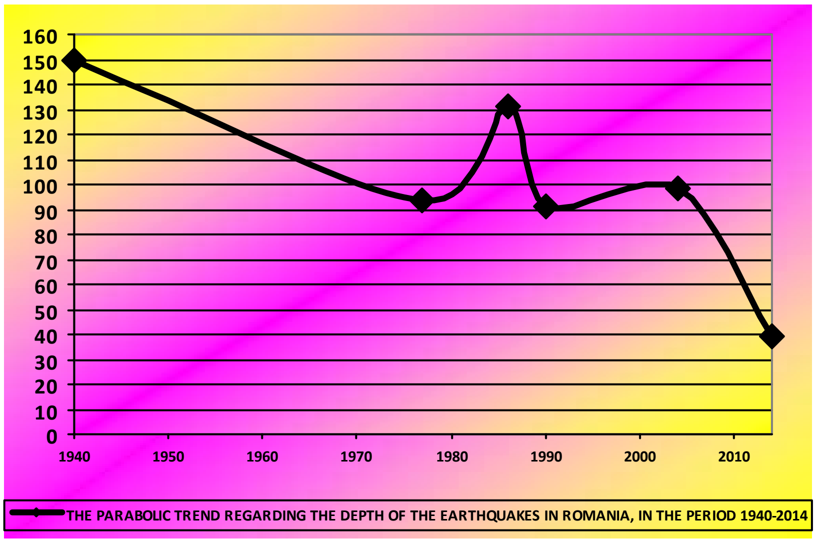

Figure 2. The trend model of the values for the depth of the earthquakes in Romania, in the period 1940-2014

We observe that, the cloud of points which reflects the values for the depth of the earthquakes from Romania, between 1940-2014, it carrying around a parabolic model of trend, according to the type no. 2.

4. The forecasting method through the „Least Squares Method"

We know that the evolution of the magnitudes concerning the earthquakes in Romania, between 1940-2014, reflects a parabolic trend of the shape $x_{t_i} = a + b \cdot t_i + ct_i^2$. So, in 2015, the magnitude concerning a possible earthquake will be:

$$MAGNITUDE_{2015}^{earthquake} = 7{,}371877263 + (-0{,}029787701) \cdot 50 + (-0{,}000557349) \cdot 50^2 = 4{,}5 \text{ degrees}$$

Also, the trend of the values regarding the depth of the earthquakes in Romania, in the period 1940-2014, is a parabolic trend of the shape $y_{t_i} = a + b \cdot t_i + ct_i^2$. Thus, in 2015, the depth concerning a possible earthquake will be:

$$DEPTH_{2015}^{earthquake} = 119{,}1308559 + (-1{,}241152726) \cdot 50 + (-0{,}013470178) \cdot 50^2 = 23{,}40km$$

4. Conclusions

We can to synthesize that in future, the evolutions regarding the magnitudes of the earthquakes in Romania, with the epicentre in Vrancea, have an unexpected character, while the fortuitous factor playes the principal role. According to the National Institute of Seismology from Bucharest, because the intensity of the earthquakes growed in the last time in Vrancea, we can to have between 15-20 of earthquakes in each month in 2015.

References

1. *Gauss C. F. - „Theoria Combinationis Observationum Erroribus Minimis Obnoxiae", Apud Henricum Dieterich Publising House, Gottingae, 1823.*
2. *Kariya T., Kurata H. - „Generalized Least Squares", John Wiley&Sons Publishing House, Hoboken, 2004.*
3. *Rao Radhakrishna C., Toutenburg H., Fieger A., Heuman C., Nittner T., Scheid S. - „Linear Models: Least Squares and Alternatives", Springer Series in Statistics, Springer-Verlag New York – Berlin Heidelberg Publishing House, New-York, 1999.*
4. *Wolberg J. - „Data Analysis Using the method of Least Squares: Extracting the Most Information from Experiments", Springer-Verlag New York – Berlin Heidelberg Publishing House, New-York, 2006.*
5. *www.emsc-csem.org*