THE ANNALS OF "DUNAREA DE JOS" UNIVERSITY OF GALATI
FASCICLE III, 2004 ISSN 1221-454X
ELECTROTEHNICS, ELECTRONICS, AUTOMATIC CONTROL, INFORMATICS

ROBUST DISCRETE-TIME ADAPTIVE SLIDING MODE WMR TRAJECTORY TRACKING AND PARAMETER IDENTIFICATION

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Abstract: In this paper a discrete-time adaptive sliding mode controller for three wheels mobile robot (WMR) is presented. The dynamical model with time-varying mass has been taken into account. The sliding-mode controller has designed on two components, corresponding to angular and linear displacement, respectively. In order to accomplish the robustness against parameter uncertainties an on-line closed loop identification scheme is proposed. To the both of them, angular and linear displacement controllers, parameter estimates depending of the robots mass, on-line updated, are used.

Keywords: discrete-time, robust adaptive sliding mode controller, wheeled mobile robot

1. INTRODUCTION

The WMR are extensively used in transportation, welding and painting operations from metallurgic and naval industries, as well as in polluted and human inaccessible places. A number of different approaches have been proposed in the literature for stabilization of WMR (Canudas de Wit and Sordalen, 1997, Canudas de Wit, Siciliano and Valavanis, 1998). The trajectory tracking and path following operations are widely used in real-time implementations, since WMR, working in closed loop, are usually required to follow an a priori planned path, with or without time dependency. The control problem of nonholonomic systems when there are model uncertainties has been widely addressed. Relatively few results have been presented about the robustness of WMR control concerning model uncertainties and external disturbances. The performing control design, using the kinematical model of the vehicle does not explicitly take into account parameters variation (robot mass and moment of inertia) and external disturbances(frictions and viscous forces).

Therefore, in certain situations the kinematic model of the WMR could be restrictive for controller design. The kinematical model is a simplified representation and does not correspond to reality of moving vehicle, which has unknown or time varying mass, frictions and wheel usages. All of these have well pointed out in Fierro and Lewis, 1997. Therefore, the dynamical model seems to be more relevant concerning uncertainties The controller design using the WMR dynamical model, where uncertainties in the robot physical parameters can be explicitly taken into account, tends to interest actual researches on this field (Coelho and Nunes 2002). The structural (parameter) and/or un-structural uncertainties in the model of the MIMO non-linear systems and the difficulties in parameter identification make necessary the design of the controller such that the closed loop robustness is achieved. It is well known that the robustness to structural, un-structural uncertainties and external disturbances of the WMR closed loop can be achieved with a variable structure controller (Aghilar, and all. 1997; Filipescu, and all 2003; Utkin, 1992;

Yu and Xu 2002). Maintaining the system on a sliding surface weakens the influence of the

This paper was recommended for publication by Emil CEANGA

uncertainties in the closed loop and quickly leads to an equilibrium point. The main advantage of the discrete-time sliding mode control is with the direct and easy real-time implementation. Since the sliding mode control is original from continuous time is more difficult to quantify when a discrete-time implementation is adopted. The discrete-time sliding mode control (Young et all, 1999; Yu and Xu, 2002) is quiet different of the accepted practice of performing the control design in the continuous-time domain. Many implementations are based on discretization, with sufficiently fast sampling rate, of the controllers designed in continuous-time. The rapid progress of the microprocessor based hardware encourages on this way. Discrete-time sliding mode controller design is usually based on an approximate sliding mode system evolution due to the non unique attractiveness condition and approximate sitting on sliding surface (Furuta 1990; Yu and Xu, 2002). The robust trajectory tracking problem has been addressed in Yang and Kim, 1999 using a continuous time sliding mode control technique, the discrete-time controller being obtained after discretization. In that approach, the control law presents singularities for specific trajectories and requires supplementary assumptions on the possible WMR motions. However, the proposed control scheme had the advantage to solve the trajectory tracking problem based on dynamical model and for non closed form of the reference trajectory. In this paper, the trajectory tracking problem for three wheels mobile robot in the presence of uncertainties (time varying mass) has been solved by means of discrete-time sliding mode control law based on the discrete-time WMR dynamical model. The asymptotic boundedness of the tracking error has been proofed. The paper is organized as follows. In Section 3 the dynamical model of three wheels mobile robot is presented. State apace model, its uncertainties and nonholonomic constraint are presented, too. The discrete-time state space model, the nonholonomic constraint and the output tracking errors of the WMR are depicted in Section 3. In Section 4 an on-line parameter identification scheme is presented. The sliding mode adaptive controller, associated to angular displacement, is designed in Section 5. Section 6 is dedicated to linear displacement sliding mode controller design. Closed loop simulation results are presented in Section 7. Some conclusion remarks from Section 8 and References can be found at end of the paper.

2. CONTINUOUS-TIME WMR DYNAMICAL MODEL

In figure 1 is shown the schema of a WMR, where XY is an inertial reference system and $X^{'}Y^{'}$ is one fixed with the vehicle. The vehicle dynamics is fully described by a three dimensional vector of generalized coordinates $q(t)$ constituted by the coordinates $((x(t), y(t)))$ of the midpoint between the two driving wheels, and by the orientation angle $\Phi(t)$ with respect to the inertial reference system

$$(1)\quad q(t) = (x(t), y(t), \Phi(t))$$

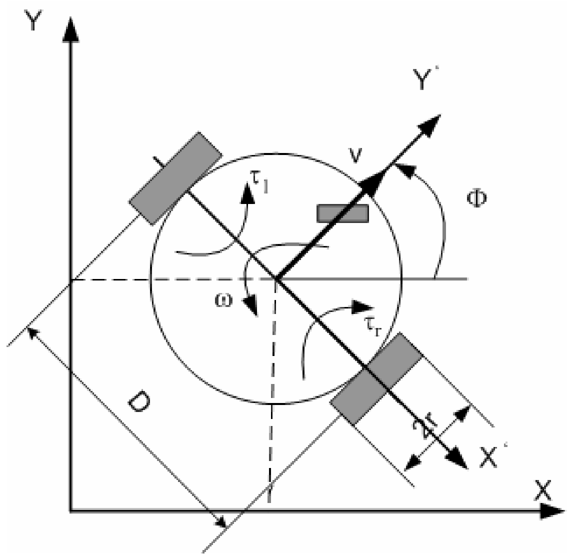

Fig.1. Three wheels mobile robotic scheme. XY inertial reference system. $X^{'}$ $Y^{'}$ fixed reference system.

The vehicle displacement is ideally subject to an independent velocity constraint of the form

$$(2)\quad \dot{x}\sin\Phi - \dot{y}\cos\Phi = 0$$

Assumption 1: The WMR displacement is supposed to be pure rolling, without of any slipping.

Define by τ_r and τ_l the torques provided by independent actuators to the right and left wheel, respectively. The vehicle is described by the following dynamical model (Fierro and Lewis, 1997)

$$(3)\quad \begin{aligned} m\ddot{x} &= -m\dot{y}\dot{\Phi} + \frac{\tau_r + \tau_l}{r}\cos\Phi \\ m\ddot{y} &= m\dot{x}\dot{\Phi} + \frac{\tau_r + \tau_l}{r}\sin\Phi \\ I\ddot{\Phi} &= \frac{D}{2r}(\tau_r - \tau_l) \end{aligned}$$

where m, I, D, r are the robot mass, moment of inertia, distance between wheels and wheels radius, respectively. The real mass of the WMR is supposed time variable with bounded uncertainty

$$(4)\quad m^{real}(t) = m^{nom} + \Delta m(t); \quad |\Delta m| \le \Delta m^{max}$$

where the nominal mass m^{nom} is known. Due to the time-varying mass, the moment of inertia is time variable, also with bounded uncertainty

$$I(t) = m(t)\frac{D^2}{4} = I^{nom} + \Delta I(t); \quad |\Delta I| \le \Delta I^{max} \tag{5}$$

Assumption 2: Even if the moment of inertia is considered time-varying, the robotic mass is supposed to be uniformly distributed all the time.

Define two parameters corresponding to the angular displacement and linear displacement

$$\alpha(t) = \frac{D}{2I(t)r} \tag{6}$$

$$\lambda(t) = \frac{1}{m(t)r} \tag{7}$$

respectively. Taking into account (4) and (5), the real values of the above defined uncertain parameters are time varying with upper bounded uncertainties

$$\begin{aligned} \alpha^{real}(t) &= \alpha^{nom} - \Delta\alpha(t); \quad |\Delta\alpha| \le \Delta\alpha^{max} \\ \lambda^{real}(t) &= \lambda^{nom} - \Delta\lambda(t); \quad |\Delta\lambda| \le \Delta\lambda^{max} \end{aligned} \tag{8}$$

Remark 1: The uncertainties could be found not only in the robotic mass and in the moment of inertia. Other parameters, like as wheel radius, distance between wheels may have uncertainties.

Let $x \in R^6$ the state vector, whose elements are

$$\begin{aligned} x_1 &= x, \quad x_2 = y, \quad x_3 = \Phi \\ x_4 &= \dot{x}, \quad x_5 = \dot{y}, \quad x_6 = \dot{\Phi} \end{aligned} \tag{9}$$

Define the control input corresponding to angular displacement

$$u_A = \tau_r - \tau_l \tag{10}$$

and the control input corresponding to linear displacement

$$u_L = \tau_r + \tau_l \tag{11}$$

With the above notations, it can write the state space model of the WMR

$$\begin{aligned} \dot{x}_1 &= x_4 \\ \dot{x}_2 &= x_5 \\ \dot{x}_3 &= x_6 \\ \dot{x}_4 &= -x_5 x_6 + \lambda^{real}(t)\cos(\Phi)u_L \\ \dot{x}_5 &= x_4 x_6 + \lambda^{real}(t)\sin(\Phi)u_L \\ \dot{x}_6 &= \alpha^{real}(t)u_A \end{aligned} \tag{12}$$

3. DISCRETE-TIME WMR DYNAMICAL MODEL

The state space representation of WMR from (12) and the non-holonomic constraint from (2) are discretized with the sampling period T, replacing the derivative by a finite difference and using a zero order hold for the control inputs.

Assumptions 3: The functions $\sin(\Phi)$ and $\cos(\Phi)$ have slow variations inside of any simple interval.

The discrete-time state space dynamical model of WMR, equivalent of (12), is

$$\begin{aligned} x_1(k+1) &= x_1(k) + Tx_4(k) \\ x_2(k+1) &= x_2(k) + Tx_5(k) \\ x_3(k+1) &= x_3(k) + Tx_6(k) \\ x_4(k+1) &= x_4(k) - Tx_5(k)x_6(k) + T\lambda(k)\cos(x_3(k))u_L(k) \\ x_5(k+1) &= x_5(k) + Tx_4(k)x_6(k) + T\lambda(k)\sin(x_3(k))u_L(k) \\ x_6(k+1) &= x_6(k) + T\alpha(k)u_A(k) \end{aligned} \tag{13}$$

k being the k-th time interval where the corresponding variable is evaluated ($t = kT$). The discrete-time equivalent of nonholonomic constraint (2) can be written as

$$x_4(k)\sin(x_3(k)) - x_5(k)\cos(x_3(k)) = 0 \tag{14}$$

Define $e(k) \in R^6$ the vector of output errors having the elements

$$e_i(k) = x_i(k) - x_i^{ref}(k); \quad i = 1, \cdots, 6 \tag{15}$$

where $x_i^{ref}(k); \quad i = 1, \cdots, 6$ is the trajectory to be tracked.

4. ON-LINE PARAMETER ESTIMATOR

Due to the time-varying of the WMR mass, the control input parameters $\alpha(t)$ and $\lambda(t)$ are on-line updated in order to be used in the corresponding sliding mode control input. The robustness against mass uncertainty will be assured by using the maximum bounds of control input parameters for angular and linear displacements to the attractiveness conditions. As will be shown in the next sections the attractiveness condition of the corresponding sliding surface is satisfied only on certain interval and outside of it, estimates of those parameters will be used to compute the control input. Moreover, in discrete-time, the sliding conditions are satisfied with some approximation and only when the system is inside of sliding sector or in the neighborhood of sliding surface, the parameter updating law can provide convergent estimates.

Let $S_A(k)$ and $S_L(k)$ be two sliding surfaces corresponding to the control input for angular and linear displacement, respectively. The on-line version of least squares method is used as parameter updating law.

The control input for angular displacement has two terms: the first one, denoted compensation part $u_A^{comp}(k)$ has to compensate the rotational dynamic; the second one denoted sliding mode part $u_A^{sm}(k)$, corresponds to system evolution inside of sliding surface neighborhood. The whole control input for angular displacement is

$$(16)\quad u_A(k) = u_A^{comp}(k) + u_A^{sm}(k)$$

The expressions and the steps for getting them, for both components of the angular displacement control input, will be provided in the next section.

If the estimated value for angular displacement control input parameters has the expression

$$(17)\quad \hat{\alpha}(k) = \alpha^{nom} - \Delta\hat{\alpha}(k)$$

then the next sequence, corresponding to recursive least squares method (Ljung, 1999; Stoica and Ahgren, 2002) can be used to provide an estimation of the uncertainty scalar term $\Delta\alpha(k)$ at the k-th step

$$(18)\quad L_{\Delta\alpha}(k) = \frac{P_{\Delta\alpha}(k-1)u_A(k-1)}{1+[u_A(k-1)]^2 P_{\Delta\alpha}(k-1)}$$

$$(19)\quad P_{\Delta\alpha}(k) = P_{\Delta\alpha}(k-1) - L_{\Delta\alpha}(k)u_A(k-1)P_{\Delta\alpha}(k-1)$$

$$(20)\quad \Delta\hat{\alpha}(k) = \Delta\hat{\alpha}(k-1) + L_{\Delta\alpha}\begin{bmatrix} \Delta\hat{\alpha}(k-1)u_A(k-1) \\ +\alpha^{nom}u_A^{sm}(k-1) \\ -\dfrac{S_A(k)}{T^2} \end{bmatrix}$$

Since just one parameter is estimated, the gain $L_{\Delta\alpha}(k)$ and the covariance $P_{\Delta\alpha}(k)$ are scalars, updated following (18) and (19), respectively. The expression of the updating law for $\Delta\alpha(k)$ will be more explained later.

Concerning the linear displacement control input parameter, the same updating law is used

$$(21)\quad \hat{\lambda}(k) = \lambda^{nom} - \Delta\hat{\lambda}(k)$$

$$(22)\quad L_{\Delta\lambda}(k) = \frac{P_{\Delta\lambda}(k-1)u_L(k-1)}{1+[u_L(k-1)]^2 P_{\Delta\lambda}(k-1)}$$

$$(23)\quad P_{\Delta\lambda}(k) = P_{\Delta\lambda}(k-1) - L_{\Delta\lambda}(k)u_L(k-1)P_{\Delta\lambda}(k-1)$$

$$(24)\quad \Delta\hat{\lambda}(k) = \Delta\hat{\lambda}(k-1) + L_{\Delta\lambda}(k)\begin{bmatrix} T\Delta\hat{\lambda}(k-1)u_L(k-1) \\ +\lambda^{nom}u_L(k-1) \\ +\tilde{S}_L(k) - S_L(k) \end{bmatrix}$$

where $\tilde{S}_L(k)$ will be defined later and $L_{\Delta\lambda}(k)$, $P_{\Delta\lambda}(k)$ have the same meaning as previously.

Remark 2: To both parameter updating laws, (20) and (24), the expression in brackets is valid when the system is in the neighborhood of the corresponding sliding surface.

5. CONTROL INPUT FOR ANGULAR DISPLACEMENT

In order to design the control input for angular displacement, the following sliding surface has been chosen

$$(25)\quad S_A(k) = A(k+1) - \mu A(k) = 0$$

where $\mu \in (-1 \quad 1)$ and

$$(26)\quad A(k) = x_3(k) - \operatorname{arctg}\left(\frac{x_5^{ref}(k) - \delta_2 e_2(k-1)}{x_4^{ref}(k) - \delta_1 e_1(k-1)}\right)$$

with $\delta_1, \delta_2 \in \left(0 \quad \frac{1}{T}\right)$. The dynamics of sliding surface is given by μ and by the position errors, e_1 and e_2. If is taken into account the non-holonomic constraint corresponding to the reference trajectory

$$(27)\quad x_3^{ref}(k) = \arctan\left(\frac{x_5^{ref}(k)}{x_4^{ref}(k)}\right)$$

then the angular error $e_3(k)$ vanish when $e_1(k)$, $e_2(k)$ tend to zero.

Remark 3: The sliding surface defined in (25) has been chosen such as whenever a sliding mode is achieved on it and $e_1(k)$, $e_2(k)$ vanish then the orientation Φ angle tends to its reference value.

The following attractiveness condition (Furuta, 1990; Yu and Xu, 2002) has been considered for computing the control input, which assures an approximate sliding mode on the surface (25)

$$S_A(k)\Delta S_A(k+1) < -\frac{1}{2}\Delta S_A(k+1) \quad (28)$$

where

$$\begin{aligned} S_A(k+1) &= A(k+2) - \mu A(k+1) \\ &= x_3(k+2) - \operatorname{arctg}\left(\frac{x_5^{ref}(k+2) - \delta_2 e_2(k+1)}{x_4^{ref}(k+2) - \delta_1 e_1(k+1)}\right) \\ &\quad - \mu\left[x_3(k+1) - \operatorname{arctg}\left(\frac{x_5^{ref}(k+1) - \delta_2 e_2(k)}{x_4^{ref}(k+1) - \delta_1 e_1(k)}\right)\right] \end{aligned} \quad (29)$$

$$\Delta S_A(k+1) = S_A(k+1) - S_A(k) \quad (30)$$

If for the compensation part of the control input is chosen the expression

$$u_A^{comp}(k) = \left(T^2\alpha^{nom}\right)^{-1} \cdot \left[\operatorname{arctg}\left(\frac{x_5^{ref}(k+2) - \delta_2 e_2(k+1)}{x_4^{ref}(k+2) - \delta_1 e_1(k+1)}\right) - x_3(k+1) - Tx_6(k) - \mu A(k+1)\right] \quad (31)$$

then, after replacing (16), (25) , (26) and (29) in (30) and some calculus manipulations, one obtains

$$\Delta S_A(k+1) = T^2\left(\alpha^{nom} - \Delta\alpha(k)\right)u_A^{sm}(k) + \Delta\alpha(k)u_A^{comp}(k) - S_A(k) \quad (32)$$

With (29) and (32), the attractiveness condition (28) becomes

$$\begin{aligned} &T^2\left[\alpha^{nom} - \Delta\alpha(k)\right]^2\left[u_A^{sm}(k)\right]^2 \\ &+ 2T^2\left[\alpha^{nom} - \Delta\alpha(k)\right]\Delta\alpha(k)\left|u_A^{sm}(k)\right|\left|u_A^{comp}(k)\right| \\ &+ T^2[\Delta\alpha(k)]^2\left[u_A^{comp}(k)\right]^2 - \left[S_A^2(k)\right]^2 < 0 \end{aligned} \quad (33)$$

With the upper bound of the angular displacement uncertainty, from (8), the above second degree inequality can be written as

$$\begin{aligned} &T^2\left[\alpha^{nom} - \Delta\alpha^{max}\right]^2\left[u_A^{sm}(k)\right]^2 \\ &+ 2T^2\left[\alpha^{nom} - \Delta\alpha^{max}\right]\Delta\alpha^{max}\left|u_A^{sm}(k)\right|\left|u_A^{comp}(k)\right| \\ &+ T^2\left[\Delta\alpha^{max}\right]^2\left[u_A^{comp}(k)\right]^2 - \left[S_A^2(k)\right]^2 < 0 \end{aligned} \quad (34)$$

as well as in the equivalent form

$$T^2\left[\begin{matrix}\left(\alpha^{nom} - \Delta\alpha^{max}\right)\left|u_A^{sm}(k)\right| \\ + \Delta\alpha^{max}\left|u_A^{comp}(k)\right|\end{matrix}\right]^2 - \left[S_A^2(k)\right]^2 < 0 \quad (35)$$

If $u_A^{sm}(k) > 0$ then (35) is equivalent with

$$\begin{aligned} 0 &< T^2\left(\alpha^{nom} - \Delta\alpha^{max}\right)u_A^{sm}(k) \\ &< -T^2\Delta\alpha^{max}\left|u_A^{comp}(k)\right| + \left|S_A(k)\right| \end{aligned} \quad (36)$$

If

$$\frac{\left|S_A(k)\right|}{T^2} > \Delta\alpha^{max}\left|u_A^{comp}(k)\right| \quad (37)$$

then, form (36), the sliding mode part of the control input can be expressed as

$$u_A^{sm}(k) < \frac{\left|\frac{S_A(k)}{T^2}\right| - \Delta\alpha^{max}\left|u_A^{comp}(k)\right|}{\alpha^{nom} - \Delta\alpha^{max}} \quad (38)$$

When $u_A^{sm}(k) < 0$, the inequality (35) is equivalent with

$$\begin{aligned} 0 &< -T^2\left(\alpha^{nom} - \Delta\alpha^{max}\right)u_A^{sm}(k) \\ &\quad + T^2\Delta\alpha^{max}\left|u_A^{comp}(k)\right| - \left|S_A(k)\right| \end{aligned} \quad (39)$$

from where

$$u_A^{sm}(k) > -\frac{\left|\frac{S_A(k)}{T^2}\right| - \Delta\alpha^{max}\left|u_A^{comp}(k)\right|}{\alpha^{nom} - \Delta\alpha^{max}} \quad (40)$$

Remark 4: The both expressions of the sliding mode component, (38) and (40), can be written by a single one

$$u_A^{sm}(k) = \rho_A \frac{\left|\frac{S_A(k)}{T^2}\right| - \Delta\alpha^{max}\left|u_A^{comp}(k)\right|}{\alpha^{nom} - \Delta\alpha^{max}} \quad (41)$$

where $\rho_A \in (-1 \quad 1)$.

When

$$\frac{|S_A(k)|}{T^2} \le \Delta\alpha^{max}\left|u_A^{comp}(k)\right| \quad (42)$$

the attractiveness condition (28) can not be satisfied. In this situation the sliding mode part of the control input can still be computed by using estimates or the corresponding parameter $\Delta\alpha$. The recursive least square method used to compute $\Delta\hat{\alpha}$, given by (18), (19) and (20), is convergent only when the system evolutes in the neighborhood of sliding surface. Therefore, when the system evolutes in a neighborhood of sliding surface an approximate sliding mode condition is satisfied

$$\frac{S_A(k+1)}{T^2} \approx 0 \quad (43)$$

The above condition is used to compute the sliding mode part of the control input for angular displacement. The relationship (43) is equivalent with

$$\left[\alpha^{nom} - \Delta\hat{\alpha}(k)\right]u_A^{sm}(k) + \Delta\hat{\alpha}(k)u_A^{comp}(k) \approx 0 \quad (44)$$

from where can be expressed

$$u_A^{sm}(k) = -\frac{\Delta\hat{\alpha}(k)u_A^{comp}(k)}{\alpha^{nom} - \Delta\hat{\alpha}(k)} \quad (45)$$

Remark 5: With the relationship (44), can be better shown the original expression of the updating law for $\Delta\alpha$

$$\begin{aligned}\Delta\hat{\alpha}(k) &= \Delta\hat{\alpha}(k-1) \\ &+ L_{\Delta\alpha}\left[\begin{array}{l}\left[\alpha^{nom} - \Delta\hat{\alpha}(k-1)\right]u_A^{sm}(k-1) \\ + \Delta\hat{\alpha}(k-1)u_A^{comp}(k-1) - \dfrac{S_A(k)}{T^2}\end{array}\right]\end{aligned} \quad (46)$$

6. CONTROL INPUT FOR LINEAR DISPLACEMET

The same steps as to control input for angular displacement will be followed to design a sliding mode control law for linear displacement.

The following sliding surface has been chosen

$$\begin{aligned}S_L(k) &= \sqrt{[x_4(k)]^2 + [x_5(k)]^2} \\ &- \sqrt{\left[x_4^{ref}(k) - \delta_1 e_1(k-1)\right]^2 + \left[x_5^{ref}(k) - \delta_2 e_1(k-1)\right]^2} \\ &= 0\end{aligned} \quad (47)$$

Starting with the third equation from the state model (13), by using the trigonometric equality

$$\begin{aligned}tg(Tx_6(k)) &= tg(x_3(k+1) - x_3(k)) \\ &= \frac{tg(x_3(k+1)) - tg(x_3(k))}{1 + tg(x_3(k+1))tg(x_3(k))}\end{aligned} \quad (48)$$

and the non-holonomic constraint (14), the following equality holds

$$tg(Tx_6(k)) = \frac{\dfrac{x_5(k+1)}{x_4(k+1)} - \dfrac{x_5(k)}{x_4(k)}}{1 + \dfrac{x_5(k+1)x_5(k)}{x_4(k+1)x_4(k)}} \quad (49)$$

Moreover, by introducing the expressions of the state variables from state model (13) in (49) and using the constraint (14), the equality (49) becomes

$$\begin{aligned}&tg(Tx_6(k))\left(\sqrt{[x_4(k)]^2 + [x_5(k)]^2} - T\lambda(k)u_L(k)\right) \\ &= Tx_6(k)\sqrt{[x_4(k)]^2 + [x_5(k)]^2}\end{aligned} \quad (50)$$

Define also, the quantity

$$\begin{aligned}\tilde{S}_L(k) &= \left[\left|\cos(Tx_6(k))\right|\right]^{-1}\sqrt{[x_4(k)]^2 + [x_5(k)]^2} \\ &- \sqrt{\left[x_4^{ref}(k+1) - \delta_1 e_1(k)\right]^2 + \left[x_5^{ref}(k+1) - \delta_2 e_2(k)\right]^2}\end{aligned} \quad (51)$$

The sliding motion on the surface (47) concerns the reduced order system of the robotic model

$$\begin{aligned}x_1(k+1) &= x_1(k) + Tx_4(k) \\ x_2(k+1) &= x_2(k) + Tx_5(k) \\ x_4(k+1) &= x_4(k) - Tx_5(k)x_6(k) + T\lambda(k)\cos(x_3(k))u_L(k) \\ x_5(k+1) &= x_5(k) + Tx_4(k)x_6(k) + T\lambda(k)\sin(x_3(k))u_L(k)\end{aligned} \quad (52)$$

Remark 6: Due to the form of the reduced order system, the control input for linear displacement has only one the sliding mode term

The same attractiveness condition (Furuta, 1990) has been considered for computing the linear displacement control input which assures an approximate sliding mode on the surface (47)

$$S_L(k)\Delta S_L(k+1) < -\frac{1}{2}\Delta S_L(k+1) \quad (53)$$

with

$$\Delta S_L(k+1) = S_L(k+1) - S_L(k) \quad (54)$$

$$S_L(k+1) = \sqrt{[x_4(k+1)]^2 + [x_5(k+1)]^2} - \sqrt{\left[x_4^{ref}(k+1) - \delta_1 e_1(k)\right]^2 + \left[x_5^{ref}(k+1) - \delta_2 e_1(k)\right]^2} \quad (55)$$

Due to the sliding evolution on (25), the angular $x_3(k)$ tends to hold the following value

$$\cos(Tx_3(k)) = \frac{x_4^{ref}(k) - \delta_1 e_1(k-1)}{\sqrt{\left[x_4^{ref}(k) - \delta_1 e_1(k-1)\right]^2 + \left[x_5^{ref}(k) - \delta_2 e_1(k-1)\right]^2}} \quad (56)$$

$$\sin(Tx_3(k)) = \frac{x_5^{ref}(k) - \delta_1 e_1(k-1)}{\sqrt{\left[x_4^{ref}(k) - \delta_1 e_1(k-1)\right]^2 + \left[x_5^{ref}(k) - \delta_2 e_1(k-1)\right]^2}} \quad (57)$$

then, by using (49), can be obtained the expression

$$[x_4(k+1)]^2 + [x_5(k+1)]^2 = \left[\sqrt{[x_4(k)]^2 + [x_5(k)]^2} - T\left(\lambda^{nom} - \Delta\lambda(k)\right)u_L(k)\right]^2 - \left[Tx_6(k)\sqrt{[x_4(k)]^2 + [x_5(k)]^2}\right]^2 \quad (58)$$

Finally, the relationship (58) can be written as

$$[x_4(k+1)]^2 + [x_5(k+1)]^2 = \frac{\left[\sqrt{[x_4(k)]^2 + [x_5(k)]^2} - T\left(\lambda^{nom} - \Delta\lambda(k)\right)u_L(k)\right]^2}{[\cos(Tx_6(k))]^2} \quad (59)$$

With above relation and (50), (54) and (53) become

$$S_L(k+1) = \tilde{S}_L(k) - T\left|\cos(Tx_6(k))\right|^{-1}\left(\lambda^{nom} - \Delta\lambda(k)\right)u_L(k) \quad (60)$$

$$\Delta S_L(k+1) = +\tilde{S}_L(k) - S_L(k) = T\left|\cos(Tx_6(k))\right|^{-1}\left(\lambda^{nom} - \Delta\lambda(k)\right)u_L(k) \quad (61)$$

Using (47) and (61), the attractiveness condition (53) leads to the inequality

$$\frac{\left(\lambda^{nom} - \Delta\lambda\right)^2 T^2}{[\cos(Tx_6(k))]^2}[u_L(k)]^2 + 2\frac{\left(\lambda^{nom} - \Delta\lambda\right)T}{\left|\cos(Tx_6(k))\right|}\tilde{S}_L(k)u_L(k) + \left[\tilde{S}_L(k)\right]^2 - [S_L(k)]^2 < 0 \quad (62)$$

With the upper bound of the linear displacement uncertainty from (8), the above second degree inequality can be written as

$$\frac{\left(\lambda^{nom} - \Delta\lambda^{max}\right)T}{\left|\cos(Tx_6(k))\right|}[u_L(k)]^2 + 2\frac{\left(\lambda^{nom} - \Delta\lambda^{max}\right)T}{\left|\cos(Tx_6(k))\right|}\left|\tilde{S}_L(k)\right|\left|u_L(k)\right| + \left[\tilde{S}_L(k)\right]^2 - [S_L(k)]^2 < 0 \quad (63)$$

and in an equivalent form

$$\left[\frac{\left(\lambda^{nom} - \Delta\lambda^{max}\right)T}{\left|\cos(Tx_6(k))\right|}\left|u_L(k)\right| + \left|\tilde{S}_L(k)\right|\right]^2 - [S_L(k)]^2 < 0 \quad (64)$$

If $u_L^{sm}(k) > 0$ then (63) is equivalent with

$$0 < \frac{\left(\lambda^{nom} - \Delta\lambda^{max}\right)T}{\left|\cos(Tx_6(k))\right|}u_L(k) < -\left|\tilde{S}_L(k)\right| + [S_L(k)]^2 \quad (65)$$

If

$$\left|S_L(k)\right| > \left|\tilde{S}_L(k)\right| \quad (66)$$

then, form (65), the sliding control input for linear displacement can be expressed as

$$u_L(k) = \rho_L \frac{S_L(k) - \left|\tilde{S}_L(k)\right|}{T\left[\left|\cos(Tx_6(k))\right|\right]^{-1}\left(\lambda^{nom} - \Delta\lambda^{max}\right)} \quad (67)$$

where $\rho_L \in (0 \quad 1)$. When

$$(68)\quad |S_L(k)| \le |\tilde{S}_L(k)|$$

the attractiveness condition (52) can not be satisfied. In this situation the control input can still be computed by using estimates of the corresponding uncertain parameter $\Delta\lambda$.

Remark 7: The recursive least square method used to compute $\Delta\hat{\lambda}$, given by (22), (23) and (24) is convergent only when the system evolutes in the neighbourhood of sliding surface. Therefore, when the system evolutes in a neighbourhood of sliding surface the approximate sliding mode condition is satisfied

$$(69)\quad S_L(k+1) \approx 0$$

The above condition is used to compute the sliding control input for linear displacement. The relationship (69) is equivalent with

$$(70)\quad T\left[\left|\cos\left(Tx_6(k)\right)\right|\right]^{-1}\left(\lambda^{nom} - \Delta\hat{\lambda}(k)\right)u_L(k) + \tilde{S}_L(k) \approx 0$$

from where the control input can be expressed

$$(71)\quad u_L(k) = -\frac{\tilde{S}_L(k)}{T\left[\left|\cos\left(Tx_6(k)\right)\right|\right]^{-1}\left(\lambda^{nom} - \Delta\hat{\lambda}(k)\right)}$$

Remark 8: Due to relationship (70), can be better shown where comes from the updating expression of $\Delta\lambda$. The original expression is

$$(72)\quad \begin{aligned}\Delta\hat{\lambda}(k) &= \Delta\hat{\lambda}(k-1) \\ &+ L_{\Delta\lambda}(k)\begin{bmatrix} T\left[\left|\cos\left(Tx_6(k-1)\right)\right|\right]^{-1}\left[\lambda^{nom} - \Delta\hat{\lambda}(k-1)\right] \\ u_L(k-1) + \tilde{S}_L(k-1) - S_L(k)\end{bmatrix}\end{aligned}$$

When the system evolutes in sliding mode on the surface (47), the followings hold

$$(73)\quad x_4(k) = x_4^{ref}(k) - \delta_1 e_1(k-1)$$

$$(74)\quad x_5(k) = x_5^{ref}(k) - \delta_2 e_1(k-1)$$

and the output tracking error dynamics associated to the reduced order system can be expressed as

$$(75)\quad e_1(k) = e_1(k) - \delta_1 T e_1(k-1)$$

$$(76)\quad e_2(k) = e_2(k) - \delta_2 T e_2(k-1)$$

For $\delta_1, \delta_2 \in \left(0 \quad \frac{1}{T}\right)$, the dynamics errors from (75) and (76) are stable. Hence, the reduced order error system is bounded during sliding mode evolution on the surface (47).

Remark 9: The reduced order system (52) may be considered after an exact sliding mode evolution on the surface (25).

7. CLOSED LOOP SIMULATION RESULTS

For testing the proposed discrete-time sliding mode adaptive controller, Scout three wheels mobile robot has been chosen. The parameters of dynamical model (3) are: m=80kg, D=0.34m, r=0.1m, I=2,312kgm^2, T=0.3s. For navigation and obstacle avoidance Scout robot has an odometric system based on incremental encoders and a sonar system based on ultrasounds. For driving each of two wheels is actuated by a DC motor. A time varying mass has been considered to the nominal one. The moment of inertia has been computed assuming the mass uniformly distributed. A linear and a constant reference trajectory have been chosen for linear and angular displacement, respectively. A linear time varying mass has been considered additionally to the nominal one. Therefore, during of simulation time the robotic real mass has been increased from 80kg to 110kg. The closed loop structure, shown in the figure 2, has been tested by simulation. The simulation, shown in figures 3 and 4, has been done for $\Delta\alpha^{max} = 0.4$ $\Delta\lambda^{max} = 0.033$, starting from the followings initial conditions: $x_1(0) = 0$, $x_2(0) = 1$, $x_3(0) = -\pi/4$, $x_4(0) = 0$, $x_5(0) = 0$, $x_6(0) = 0$ .The following values have been chosen for the constants: $\mu = 0.001$, $\rho_L = \rho_A = 0.99$, $\delta_1 = \delta_2 = 0.99$, $P_{\Delta\alpha}(0) = P_{\Delta\lambda}(0) = 10$.

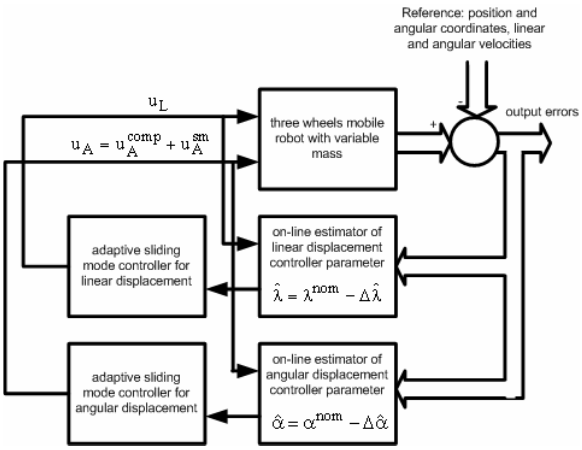

Fig. 2. Closed loop block schema of sliding mode adaptive wheeled mobile robot control

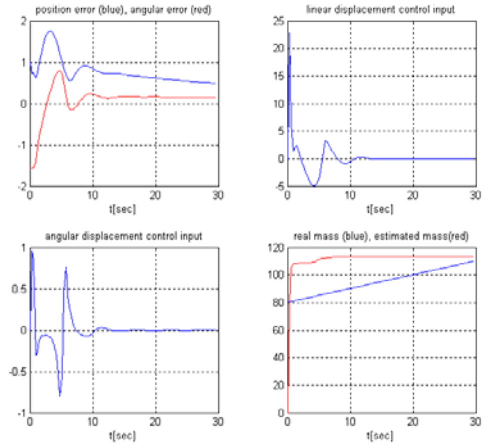

Fig. 3. Angular and positions errors, control inputs, real and estimated mass.

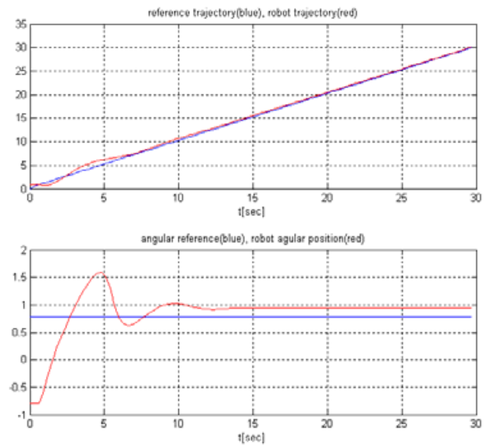

Fig. 4. Linear and angular trajectories tracking.

8. CONCLUSIONS

A discrete-time sliding mode adaptive controller for trajectory tracking three wheels mobile robots has presented in this paper. The time-varying mass dynamical state space model has undertaken in order to design the controller. Even if as model uncertainty only the robotic mass has been considered, the proposed controller assures closed loop robustness to a wide typology of model uncertainties and external disturbances. Two components of the sliding mode adaptive controller have been designed, for angular and linear displacement, respectively. The robustness is guaranteed by sliding mode controller and by an adaptive parameter identification scheme. Controller parameters, on-line updated, assure an approximate sliding mode evolution even if the attractiveness condition is not satisfied and, moreover contribute to an increased robustness. Closed loop simulations with three wheels Scout mobile robot have presented. Discrete time dynamical model and controller design lead to an easy real-time implementation.

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