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SLIDING-MODE ADAPTIVE CONTROL OF PIONEER 3-DX WHEELED MOBILE ROBOT

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Abstract: Parameter identification scheme and discrete-time adaptive sliding-mode controller applied to Pioneer 3-DX wheeled mobile robot (WMR) are presented in this paper. The dynamical model for mobile robot with one pair of active wheels, time–varying mass and moment of inertia have been used in sliding-mode control. Two closed-loop, on-line parameter estimators have been used in order to achieve robustness against parameter uncertainties (robot mass and moment of inertia). Two sliding-mode adaptive controllers corresponding to angular and position motion have been designed. Closed-loop circular trajectory tracking Pioneer 3-DX real-time control is presented.

Keywords: Discrete-time Pioneer 3-DX model, sliding-mode adaptive control, on-line parameter estimation.

1. INTRODUCTION

Different approaches have been proposed in the literature for output tracking of one pair of active wheels mobile robots (WMR), (Canudas de Wit and Sordalen, 1997, Canudas de Wit, Siciliano and Valavanis, 1998). The control problem of non-holonomic systems when there are model uncertainties has been widely addressed. Relatively few results have been presented about the robustness of WMR control concerning model uncertainties and external disturbances. The structural (parameter) and/or un-structural uncertainties in the model of the MIMO non-linear systems and the difficulties in parameter identification make necessary the design of the controller such that the closed loop robustness is achieved. It is well known that the robustness to structural, un-structural uncertainties and external disturbances of the WMR closed loop can be achieved with a variable structure controller, (Aghilar, and all. 1997; Filipescu, and all 2005; Yu and Xu 2002). Maintaining the system on a sliding surface weakens the influence of the uncertainties in the closed loop and quickly leads to an equilibrium point. The main advantage of the discrete-time sliding mode control is with the direct and easy real-time implementation. Since the sliding mode control is original from continuous time, it is more difficult to choose a synthesis in discrete-time. The discrete-time sliding mode control, (Yu and Xu 2002, Leo and Orlando 1998) , is quite different of performing the control design in the continuous-time domain. Discrete-time sliding-mode controller design is usually based on an approximate sliding-mode system evolution due to the non unique attractiveness condition and approximate evolution on sliding surface, (Furuta 1990; Yu and Xu, 2002). . The robust trajectory tracking problem has been addressed in Yang and Kim, 1999, using a continuous time sliding-mode control. The performing control design, using the kinematical model of the vehicle does not explicitly take into account parameters variation (robot mass and moment of inertia) and external disturbances (frictions and viscous forces), (Fierro and Lewis, 1997). The controller design using the WMR dynamical model, where uncertainties in the robot physical parameters can be explicitly taken into account, tends to interest actual researches on this field. In this paper, the trajectory tracking problem for Pioneer 3-DX one pair of active wheels type WMR, in the presence of uncertainties (time-varying

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mass and moment of inertia), has been solved by discrete-time sliding-mode controllers based on the discrete-time WMR dynamical model. Two closed loop, on-line parameter estimators have been used against parameter uncertainties.

The paper is organized as follows. In Section 2 the dynamical model of one pair of active wheels Pioneer 3-DX mobile robot is presented. Also, the discrete-time state space model, its uncertainties, non-holonomic constraint and the output tracking errors of Pioneer 3-DX are presented. Section 3 describes on-line parameter estimators corresponding to angular and position motion. The sliding adaptive controllers, associated to angular and position motion, are designed in Section 4 and 5. Pioneer 3-DX sliding-mode closed loop real-time results are presented in Section 6 and conclusions remarks in Section 7.

2. PIONEER 3-DX DYNAMIC MODEL

1) Assumption: The WMR motion is supposed to be pure rolling, without of any slipping.

Figures 1 and 2 show Pioneer 3-DX with Pioneer 5-DOF manipulator and the schema of a WMR, respectively. The vehicle dynamics is fully described by a three dimensional vector of generalized coordinates $q(t)$ constituted by the coordinates $((x(t), y(t)))$ of the midpoint between the two driving wheels, and by the orientation angle $\Phi(t)$. The velocity constraint (non-holonomic constraint) of vehicle motion is $\dot{x}\sin\Phi - \dot{y}\cos\Phi = 0$. Define by τ_r and τ_l the torques provided by DC motors to the right and left wheel, respectively. The vehicle is described by the following dynamical model where m, I, D, r are the robot mass, moment of inertia, distance between wheels and wheels radius, respectively

$$\begin{aligned} m\ddot{x} &= -m\dot{y}\dot{\Phi} + \frac{\tau_r + \tau_l}{r}\cos\Phi \\ m\ddot{y} &= m\dot{x}\dot{\Phi} + \frac{\tau_r + \tau_l}{r}\sin\Phi \\ I\ddot{\Phi} &= \frac{D}{2r}(\tau_r - \tau_l) \end{aligned} \tag{1}$$

The real mass of the WMR is supposed to be time-varying with bounded uncertainty with known nominal mass. Due to the time-varying mass, the moment of inertia becomes time-depending with bounded uncertainty.

2) Assumption: Even if the moment of inertia is considered time-varying, the robotic mass is supposed to be uniformly distributed all the time.

Fig. 1. Pioneer 3-DX with 5-DOF robotic manipulator

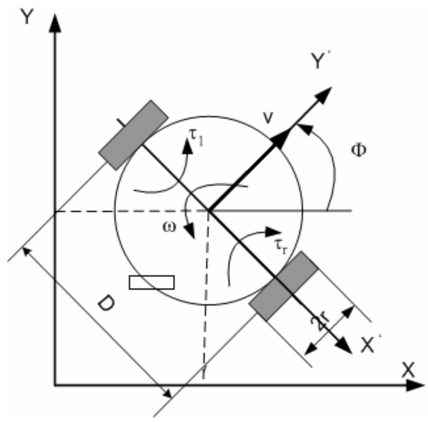

Fig. 2. WMR configuration variables for angular and position motion.

Let define two parameters corresponding to the angular and position motion, such as: $\alpha(t) = D/(2I(t)r)$, $\pi(t) = 1/(m(t)r)$. The real values of the parameters are time-varying with upper bounded uncertainties

$$\begin{aligned} \alpha^{real}(t) &= \alpha^{nom} - \Delta\alpha(t); \quad |\Delta\alpha| \le \Delta\alpha^{\max} \\ \pi^{real}(t) &= \pi^{nom} - \Delta\pi(t); \quad |\Delta\pi| \le \Delta\pi^{\max} \end{aligned} \tag{2}$$

Let $x \in R^6$ be the state vector, whose elements are

$$\begin{aligned} x_1 &= x, \quad x_2 = y, \quad x_3 = \Phi \\ x_4 &= \dot{x}, \quad x_5 = \dot{y}, \quad x_6 = \dot{\Phi} \end{aligned} \tag{3}$$

Define the control input corresponding to angular, $u_A = \tau_r - \tau_l$ and position motion, $u_P = \tau_r + \tau_l$, respectively. The state space representation of WMR and the non-holonomic constraint will be discretized with the sampling period T, replacing the derivative by a finite difference and using a zero-order-hold for the control inputs, k being the k^{th} time interval where the corresponding variable is evaluated ($t = kT$). Let $e(k) \in R^6$ be, the vector of output errors: $e_i(k) = x_i(k) - x_i^{ref}(k)$, $x_i^{ref}(k)$; $i = 1, \cdots, 6$ is the trajectory to be tracked.

$$\begin{aligned} x_1(k+1) &= x_1(k) + Tx_4(k) \\ x_2(k+1) &= x_2(k) + Tx_5(k) \\ x_3(k+1) &= x_3(k) + Tx_6(k) \\ x_4(k+1) &= x_4(k) - Tx_5(k)x_6(k) + T\pi(k)\cos(x_3(k))u_P(k) \\ x_5(k+1) &= x_5(k) + Tx_4(k)x_6(k) + T\pi(k)\sin(x_3(k))u_P(k) \\ x_6(k+1) &= x_6(k) + T\alpha(k)u_A(k) \end{aligned} \tag{4}$$

$$x_4(k)\sin(x_3(k)) - x_5(k)\cos(x_3(k)) = 0 \tag{5}$$

3. ANGULAR AND POSITION MOTION ON-LINE PARAMETER ESTIMATION

The closed loop structure, shown in figure 2, is proposed. For each robot motion, angular and position, respectively, an on-line parameter estimator and a sliding controller have been introduced. Due to the time-varying of the Pioneer 3-DX mass, the control input parameters $\alpha(t)$ and $\pi(t)$ are on-line updated in order to be used in the corresponding sliding mode control input. The robustness against mass uncertainty will be assured. The maximum bounds of control input parameters corresponding to angular and linear motion will be used in the attractiveness condition of appropriate sliding surface. As will be shown in the next sections, the attractiveness condition of the corresponding sliding surface only on certain interval is satisfied. Outside of it, on-line parameter estimates will be used to compute the control input. Moreover, in discrete-time, the sliding condition with some approximation is satisfied. When the system is inside of the sliding sector or in the neighborhood of sliding surface, the parameter updating law can provide convergent estimates. Let $S_A(k)$ and $S_P(k)$ be two sliding surfaces corresponding to the control input for angular and position motion, respectively. As parameter updating law, the recursive least squares method is used. The control input for angular motion has two terms: the first one, denoted compensation part $u_A^{comp}(k)$, has to compensate the rotational dynamics; the second one, denoted sliding mode part, $u_A^{sm}(k)$, corresponds to system evolution inside of sliding surface neighborhood. The whole control input for angular motion is

$$u_A(k) = u_A^{comp}(k) + u_A^{sm}(k) \tag{6}$$

The calculus and the steps for getting both components of the angular motion control input are given in Section 4. Expressing the estimated value for angular motion control input parameter, $\hat{\alpha}(k) = \alpha^{nom} - \Delta\hat{\alpha}(k)$, the next sequence, corresponding to recursive least squares method, (Ljung, 1999; Stoica and Ahgren, 2002), can be used to provide an estimation of the uncertainty scalar term $\Delta\alpha(k)$ at the k[th] step

$$L_{\Delta\alpha}(k) = \frac{P_{\Delta\alpha}(k-1)u_A(k-1)}{1 + [u_A(k-1)]^2 P_{\Delta\alpha}(k-1)} \tag{7}$$

$$P_{\Delta\alpha}(k) = P_{\Delta\alpha}(k-1) - L_{\Delta\alpha}(k)u_A(k-1)P_{\Delta\alpha}(k-1) \tag{8}$$

$$P_{\Delta\alpha}(k) = P_{\Delta\alpha}(k-1) - L_{\Delta\alpha}(k)u_A(k-1)P_{\Delta\alpha}(k-1) \tag{9}$$

1) Remark: Since for each robot motion just one parameter is estimated, the gain $L_{\Delta\alpha}(k)$ and the covariance $P_{\Delta\alpha}(k)$ are scalars.

The control input for position motion, $u_P(k)$, has only sliding-mode part, $u_P(k) = u_P^{sm}(k)$. For the corresponding parameter, $\hat{\pi}(k) = \pi^{nom} - \Delta\hat{\pi}(k)$, similar updating law is used

$$L_{\Delta\pi}(k) = \frac{P_{\Delta\pi}(k-1)u_P(k-1)}{1 + [u_P(k-1)]^2 P_{\Delta\pi}(k-1)} \tag{10}$$

$$P_{\Delta\pi}(k) = P_{\Delta\pi}(k-1) - L_{\Delta\pi}(k)u_\pi(k-1)P_{\Delta\pi}(k-1) \tag{11}$$

$$\Delta\hat{\pi}(k) = \Delta\hat{\pi}(k-1) + L_{\Delta\pi}(k)\begin{bmatrix} T\Delta\hat{\pi}(k-1)u_P(k-1) \\ +\pi^{nom}u_P(k-1) \\ +\tilde{S}_P(k) - S_P(k) \end{bmatrix} \tag{12}$$

where $L_{\Delta\pi}(k)$, $P_{\Delta\pi}(k)$ have the same meaning as previously and $\tilde{S}_P(k)$ will be defined later.

2) Remark: For both parameter updating laws, (9) and (12), the expression in brackets is valid when the system evolutes in the neighborhood of the corresponding sliding surface.

4. ANGULAR MOTION SLIDING-MODE ADAPTIVE CONTROL

The following stable sliding surface has been chosen, in order to design the control input for angular motion

$$S_A(k) = A(k+1) - \mu A(k) = 0 \tag{13}$$

where

$$A(k) = x_3(k) - arctg\left(\frac{x_5^{ref}(k) - \delta_2 e_2(k-1)}{x_4^{ref}(k) - \delta_1 e_1(k-1)}\right) \tag{14}$$

with: $\mu \in (-1 \quad 1)$, $\delta_1, \delta_2 \in \left(0 \quad \frac{1}{T}\right)$. Parameter μ and the position errors, e_1, e_2, establish the dynamics of sliding surface. The interval set of δ_1 and δ_2 assures the stability of position errors. If the non-holonomic constraint corresponding to the reference trajectory

$$x_3^{ref}(k) = arctg\left(x_5^{ref}(k) / x_4^{ref}(k)\right) \quad (15)$$

is taken into account, then the angular error $e_3(k)$ vanish when $e_1(k)$, $e_2(k)$ tend to zero.

3) Remark: The sliding surface defined in (13) has been chosen such as whenever a sliding mode is achieved on it and $e_1(k), e_2(k)$ vanish, the orientation angle Φ tends to its reference value.

For computing the control input, the following attractiveness condition, (Furuta, 1990; Yu and Xu, 2002), has been used:

$$S_A(k)\Delta S_A(k+1) < -\frac{1}{2}\Delta S_A^2(k+1) \quad (16)$$

where

$$\Delta S_A(k+1) = S_A(k+1) - S_A(k) \quad (17)$$

An approximate sliding-mode evolution can be assured on the surface (13). If for the compensation part of the control input the expression

$$u_A^{comp}(k) = \left(T^2\alpha^{nom}\right)^{-1} \left[arctg\left(\frac{x_5^{ref}(k+2) - \delta_2 e_2(k+1)}{x_4^{ref}(k+2) - \delta_1 e_1(k+1)}\right) + x_3(k+1) - Tx_6(k) - \mu A(k+1) \right] \quad (18)$$

is chosen, then, after replacing (6), (13) and (14) in (17), one obtains

$$\Delta S_A(k+1) = T^2\left(\alpha^{nom} - \Delta\alpha(k)\right)u_A^{sm}(k) + \Delta\alpha(k)u_A^{comp}(k) - S_A(k) \quad (19)$$

With (19), (16) becomes

$$T^2\left[\alpha^{nom} - \Delta\alpha(k)\right]^2\left[u_A^{sm}(k)\right]^2 + 2T^2\left[\alpha^{nom} - \Delta\alpha(k)\right]\Delta\alpha(k)\left|u_A^{sm}(k)\right|\left|u_A^{comp}(k)\right| + T^2[\Delta\alpha(k)]^2\left[u_A^{comp}(k)\right]^2 - \left[S_A^2(k)\right]^2 < 0 \quad (20)$$

Introducing the upper bound of the angular motion parameter uncertainty, the above second degree inequality can be written in the compact form

$$T^2\left[\left(\alpha^{nom} - \Delta\alpha^{max}\right)\left|u_A^{sm}(k)\right| + \Delta\alpha^{max}\left|u_A^{comp}(k)\right|\right]^2 - \left[S_A^2(k)\right]^2 < 0 \quad (21)$$

If $u_A^{sm}(k) > 0$ and $\left|S_A(k)\right|/T^2 > \Delta\alpha^{max}\left|u_A^{comp}(k)\right|$, then the sliding-mode part of the control input can be expressed as

$$u_A^{sm}(k) < \left|S_A(k)/T^2\right| - \frac{\Delta\alpha^{max}\left|u_A^{comp}(k)\right|}{\alpha^{nom} - \Delta\alpha^{max}} \quad (22)$$

When $u_A^{sm}(k) < 0$, the inequality (21) is satisfied for

$$u_A^{sm}(k) > -\frac{\left|S_A(k)/T^2\right| - \Delta\alpha^{max}\left|u_A^{comp}(k)\right|}{\alpha^{nom} - \Delta\alpha^{max}} \quad (23)$$

3) Remark: Both expressions of the sliding-mode part, (22) and (23), can be written compactly

$$u_A^{sm}(k) = \frac{\rho_A\left|\frac{S_A(k)}{T^2}\right| - \Delta\alpha^{max}\left|u_A^{comp}(k)\right|}{\alpha^{nom} - \Delta\alpha^{max}} \quad (24)$$

where $\rho_A \in (-1 \quad 1)$.

When $\left|S_A(k)\right|/T^2 \le \Delta\alpha^{max}\left|u_A^{comp}(k)\right|$, the attractiveness condition (16) can not be satisfied. The sliding mode part of the control input still can be computed by using estimates of parameter $\Delta\alpha$. The recursive least square method used to compute $\Delta\hat{\alpha}$, given by (7), (8) and (9), is convergent only when the system evolves in the neighborhood of sliding surface. Therefore, an approximate sliding mode condition is satisfied $S_A(k+1)/T^2 \approx 0$

$$\left[\alpha^{nom} - \Delta\hat{\alpha}(k)\right]u_A^{sm}(k) + \Delta\hat{\alpha}(k)u_A^{comp}(k) \approx 0 \quad (25)$$

This approximate is used in order to compute the control input for angular motion

$$u_A^{sm}(k) = -\Delta\hat{\alpha}(k)u_A^{comp}(k) / \left(\alpha^{nom} - \Delta\hat{\alpha}(k)\right) \quad (26)$$

4) Remark: Using (24), the updating law (9) can be rewritten as

$$\Delta\hat{\alpha}(k) = \Delta\hat{\alpha}(k-1) + L_{\Delta\alpha}\left[\left[\alpha^{nom} - \Delta\hat{\alpha}(k-1)\right]u_A^{sm}(k-1) + \Delta\hat{\alpha}(k-1)u_A^{comp}(k-1) - S_A(k)/T^2\right] \quad (27)$$

5. POSITION MOTION SLIDING-MODE ADAPTIVE CONTROL

The following sliding surface is proposed

$$S_P(k) = \left([x_4(k)]^2 + [x_5(k)]^2\right)^{1/2} - \left(\left[x_4^{ref}(k) - \delta_1 e_1(k-1)\right]^2 + \left[x_5^{ref}(k) - \delta_2 e_1(k-1)\right]^2\right)^{1/2} = 0 \quad (28)$$

Starting with the third equation of model (4), using a trigonometric equality and the non-holonomic constraint (5), the following equality holds

$$tg(Tx_6(k)) = \left(\frac{x_5(k+1)}{x_4(k+1)} - \frac{x_5(k)}{x_4(k)}\right) \Bigg/ \left(1 + \frac{x_5(k+1)x_5(k)}{x_4(k+1)x_4(k)}\right) \quad (29)$$

Moreover, introducing the expressions of the state variables, from state model (4), and using the constraint (5), the above equality becomes

$$tg\left(Tx_6(k)\right)\left(\left([x_4(k)]^2+[x_5(k)]^2\right)^{1/2}-T\pi(k)u_P(k)\right)$$
$$=Tx_6(k)\left([x_4(k)]^2+[x_5(k)]^2\right)^{1/2} \quad (30)$$

Let define

$$\tilde{S}_P(k)=\left[\left|\cos(Tx_6(k))\right|\right]^{-1}\left([x_4(k)]^2+[x_5(k)]^2\right)^{1/2} -\begin{pmatrix}\left[x_4^{ref}(k+1)-\delta_1 e_1(k)\right]^2\\ +\left[x_5^{ref}(k+1)-\delta_2 e_2(k)\right]^2\end{pmatrix}^{1/2} \quad (31)$$

The sliding motion on the surface (27) concerns the reduced order system of the robotic model, without of 3rd and 6th equation. The same attractiveness condition, as in [6], for computing the position motion control input has been considered

$$S_P(k)\Delta S_P(k+1)<-\frac{1}{2}\Delta S_P^2(k+1) \quad (32)$$

$$\Delta S_P(k+1)=S_P(k+1)-S_P(k) \quad (33)$$

An approximate sliding mode evolution on the surface (27) can be assured. Consequently of sliding-mode evolution on (13), the angular state $x_3(k)$ tends to hold the following expressions

$$\cos(Tx_3(k))=\left(x_4^{ref}(k)-\delta_1 e_1(k-1)\right)\left(\left[x_4^{ref}(k)-\delta_1 e_1(k-1)\right]^2+\left[x_5^{ref}(k)-\delta_2 e_1(k-1)\right]^2\right)^{-1/2} \quad (34)$$

$$\sin(Tx_3(k))=\left(x_5^{ref}(k)-\delta_1 e_1(k-1)\right)\left(\left[x_4^{ref}(k)-\delta_1 e_1(k-1)\right]^2+\left[x_5^{ref}(k)-\delta_2 e_1(k-1)\right]^2\right)^{-1/2} \quad (35)$$

Using (28), the following expression can be obtained

$$[x_4(k+1)]^2+[x_5(k+1)]^2=[\cos(Tx_6(k))]^{-2}\left[\left([x_4(k)]^2+[x_5(k)]^2\right)^{1/2}-T\left(\pi^{nom}-\Delta\pi(k)\right)u_P(k)\right]^2 \quad (36)$$

With (35) and (29), (25) and (32) become

$$S_P(k+1)=\tilde{S}_P(k)-T\left|\cos(Tx_6(k))\right|^{-1}\left(\pi^{nom}-\Delta\pi(k)\right)u_P(k) \quad (37)$$

$$\Delta S_P(k+1)=\tilde{S}_P(k)-S_P(k)=T\left|\cos(Tx_6(k))\right|^{-1}\left(\pi^{nom}-\Delta\pi(k)\right)u_P(k) \quad (38)$$

Using (36), (37) and upper bound of position motion uncertainty, from (2), the second degree inequality can be written

$$\left[\frac{\left(\pi^{nom}-\Delta\pi^{\max}\right)T}{\left|\cos(Tx_6(k))\right|}\left|u_P(k)\right|+\left|\tilde{S}_P(k)\right|\right]^2-[S_P(k)]^2<0 \quad (39)$$

If $u_P^{sm}(k)>0$ and $\left|S_P(k)\right|>\left|\tilde{S}_P(k)\right|$, then the sliding control input for position motion is

$$u_P(k)=\rho_P\frac{S_P(k)-\left|\tilde{S}_P(k)\right|}{T\left[\left|\cos(Tx_6(k))\right|\right]^{-1}\left(\pi^{nom}-\Delta\pi^{\max}\right)} \quad (40)$$

where $\rho_P\in(0\quad 1)$. When $\left|S_P(k)\right|\le\left|\tilde{S}_P(k)\right|$, the attractiveness condition (31) can not be satisfied. The control input still can be computed using on-line estimates for $\Delta\pi$.

5) Remark: The recursive least square method used to compute $\Delta\hat{\pi}$, given by (10), (11) and (12), is convergent only when the system evolves in the neighborhood of sliding surface. Therefore, the approximate sliding mode condition is satisfied, $S_P(k+1)\approx 0$, i.e.

$$T\left[\left|\cos(Tx_6(k))\right|\right]^{-1}\left(\pi^{nom}-\Delta\hat{\pi}(k)\right)u_P(k)+\tilde{S}_P(k)\approx 0 \quad (41)$$

From above, the control input can be expressed as

$$u_P(k)=-\tilde{S}_P(k)/T\left[\left|\cos(Tx_6(k))\right|\right]^{-1}\left(\pi^{nom}-\Delta\hat{\pi}(k)\right) \quad (42)$$

6) Remark: As result of (40), (12) can be rewritten as

$$\Delta\hat{\pi}(k)=\Delta\hat{\pi}(k-1)+L_{\Delta\pi}(k)\begin{bmatrix}T\left[\left|\cos(Tx_6(k-1))\right|\right]^{-1}\left[\pi^{nom}-\Delta\hat{\pi}(k-1)\right]\\ u_P(k-1)+\tilde{S}_P(k-1)-S_P(k)\end{bmatrix} \quad (43)$$

When the system evolves in sliding-mode on the surface (27), can express the followings

$$x_4(k)=x_4^{ref}(k)-\delta_1 e_1(k-1) \quad (44)$$

$$x_5(k)=x_5^{ref}(k)-\delta_2 e_1(k-1) \quad (45)$$

Therefore, output tracking error dynamics associated to the reduced order system can be expressed as:

$$e_1(k+1)=e_1(k)-\delta_1 Te_1(k-1) \quad (46)$$

$$e_2(k+1)=e_2(k)-\delta_2 Te_2(k-1) \quad (47)$$

For $\delta_1,\delta_2\in\left(0\quad\frac{1}{T}\right)$, the above dynamics errors are stable.

6. PIONEER 3-DX REAL-TIME SLIDING-MODE CLOSED LOOP CONTROL

For testing the proposed discrete-time sliding-mode adaptive controller Pioneer 3-DX with on board PC and wireless adapter has been used in circular trajectory tracking. The rugged P3-DX is 44cm x 38cm x 22cm aluminum body with 16.5cm diametric drive wheels. The two motors use 38.3:1 gear ratios and contain 500-tick encoders. This differential drive platform is highly holonomic and can rotate in place moving both wheels, or it can swing around a stationery wheel in a circle of 32cm radius. A rear caster balances the robot. The following parameters of model (3) were used: m=10kg, D=50cm, I=0,0624 kgm^2, T=0.3s. The moment of inertia has been computed assuming the mass uniformly distributed. A linear-time varying mass additionally to the nominal one has been considered. More precisely, the robotic time-varying mass has been increased linearly from 12kg to 16kg. The circle trajectory tracking, shown in figures 3, was obtained for $\Delta\alpha^{\max} = 0.4$, $\Delta\pi^{\max} = 0.033$. The following values have been chosen for the constants: $\mu = 0.001$, $\rho_P = \rho_A = 0.99$, $\delta_1 = \delta_2 = 3.33$, $P_{\Delta\alpha}(0) = P_{\Delta\pi}(0) = 10$.

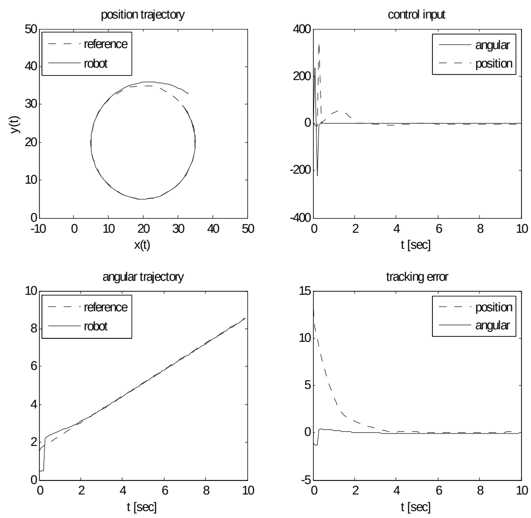

Fig.3. WMR closed loop response for circular reference and initial conditions x1(0)=33; x2(0)=33; x3(0)=π/7; x4(0)=-0.5; x5(0)=0.2; x6(0)=0.1.

7. CONCLUSION

Discrete-time, sliding-mode adaptive controllers and parameter estimators for trajectory tracking applied to control angular and position motion of Pioneer 3-DX one pair of active wheels mobile robot, have presented in this paper. The time-varying mass and moment of inertia dynamical state space model have been undertaken in order to design the controllers. Even if as parameter uncertainties, only the robotic mass and moment of inertia have been considered, the proposed controllers assure closed loop robustness to a wide typology of parameter and model uncertainties and external disturbances. Two sliding-mode adaptive controllers have been designed, for angular and position motion, respectively. The robustness is guaranteed by sliding-mode controllers and by on-line parameter estimators. Controllers parameters, on-line updated, assure an approximate sliding-mode evolution even if the attractiveness condition is not satisfied and contribute to an increased robustness.

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