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OPTIMAL CONTROL FOR ELECTRIC VEHICLE STABILIZATION

Marian Gaiceanu, Elena Voncila, Razvan Buhosu

Dunarea de Jos University of Galati
e-mail: Marian.Gaiceanu@ugal.ro

Abstract: This main objective of the paper is to stabilize an electric vehicle in optimal manner to a step lane change maneuver. To define the mathematical model of the vehicle, the rigid body moving on a plane is taken into account. An optimal lane keeping controller delivers the adequate angles in order to stabilize the vehicle's trajectory in an optimal way. Two degree of freedom linear bicycle model is adopted as vehicle model, consisting of lateral and yaw motion equations. The proposed control maintains the lateral stability by taking the feedback information from the vehicle transducers. In this way only the lateral vehicle's dynamics are enough to considerate. Based on the obtained linear mathematical model the quadratic optimal control is designed in order to maintain the lateral stability of the electric vehicle. The numerical simulation results demonstrate the feasibility of the proposed solution.

Keywords: electric vehicle, optimal control, lane keeping control, Riccati.

1. INTRODUCTION

Intelligent transportation systems have been extensively studied beginning with 1990's (Ackermann *et al.,* 1995), (Patwardhan *et al.,* 1997), (Wang *et al.,* 1999), (Yamamoto *et al.,* 1999), (Hedrick *et al.,* 1994).

Modern control has been implying in order to obtain higher performances of the vehicle response (Tanaka *et al.*, 2000), (Tai *et al.*, 2000), (Lu *et al.*, 2002), (Ibaraki *et al.*, 2005), (Jin-Hua *et al.*, 2005), (Mammar *et al.*, 2006), (Mouri *et al.*, 2002), (Feng *et al.*, 2010).

These systems have certain advantages, most importantly being the safety of the driver and passengers.

Steering control involve both techniques: lane keeping control and lane changing control (Jin-Hua *et al.,* 2005).

The main difference between the above mentioned techniques is that the later must follow a given reference input for lateral motion, instead of following the center lane, like in the first method.

Both methods put in control the vehicle in case of crosswinds, a front-tire blowout or for uneven pavement. For a step lane trajectory reference changing, the mathematical model of the lateral vehicle is used. In order to give more confidence and safety for driver and passengers, an asymptotically stable vehicle system response for all state variables is obtained by using a linear quadratic controller.

This paper was recommended for publication by Toader Munteanu

2. MATHEMATICAL MODELLING OF THE ELECTRIC VEHICLE MOVEMENT

It is well known that the conventional control based only the front–wheel steering vehicle (2WS) regulates with good performances the lateral vehicle deviation.

Two wheel steering control (2WS) has the disadvantage of damping behavior of yaw dynamics (Mouri *et al.,* 1997), (Raksincharoensak *et al.,* 2002). Therefore, in order to obtain a good damping behavior of both lateral and yaw motion, 4WS mathematical model of vehicle has been taken into consideration.

Vehicle dynamics are accurately described by 6 degrees of freedom, but in order to meet the objective of the paper only 2 degree of freedom (2DOF) are considerate to describe the lateral dynamics. This model gives a very good approximation of experimental results for the vehicle lateral dynamics (Cerone, et al., 2002).

Following the mathematical modeling of the electric drives vector control, by using the similitude principle, mathematical modeling of electric vehicle is accomplish in moving reference frame solidar with vehicle movement. In this way, moments of inertia for vehicle are constant. The vehicle motion is assumed planar.

The motion of the vehicle is described by using a three coordinates system (see Fig.1, according to Automotive Society of Engineers – SAE): x-axis is oriented on forward direction and on the longitudinal plane of symmetry of the vehicle; y-axis is oriented on lateral direction, on right side of vehicle; z-axis is oriented downward to vehicle. The origin of the coordinate system is adopted as the vehicle's center of gravity (CG).

Thereafter, taking into consideration the small angular changes, the parameters of electric vehicle movement are constant also in fixed reference system, conducting towards an invariant dynamic system.

The analysis of vehicle movement is done by neglecting the wheel suspension dynamics, effects of rolling and pitching, therefore the rigid body assumption for the vehicle is fitted.

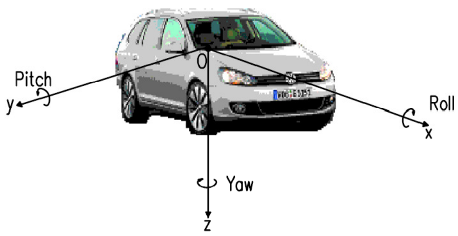

Fig.1. The standard coordinate system for the electric vehicle, according to SAE

In Fig. 2 both electric vehicle movement analysis in mobile (x,y) and fixed (α,β) coordinate systems at successive instants *t* and *t*+Δ*t* is presented.

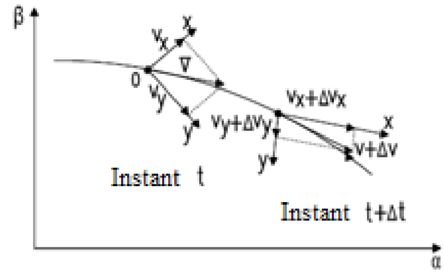

Fig.2. Vehicle movement analysis both in (x,y) and fixed (α,β) coordinate systems at successive instants *t* and *t*+Δ*t*

In order to deduct the vehicle motion equation, it is necessary to know the acceleration expression of the center of gravity of vehicle. Further, it is assumed the uniform motion of vehicle, therefore longitudinal velocity component $v_x = \dot{x} = ct$. In this way, only the lateral and yaw motion equations are necessary to define the mathematical model of the vehicle.

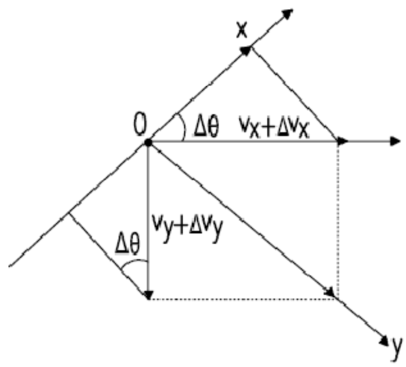

Fig.3. The variations of longitudinal and lateral velocity vehicle components

The variation of lateral velocity component, $v_y + \Delta v_y$ is shown in Fig. 3, where $\Delta\theta$ is small angular variation.

Taking into consideration the small angular variations, the following expression of the lateral acceleration component is calculated:

$$(1)\quad a_y = \dot{v}_y + v_x \dot{\theta} = \ddot{y} + \dot{x} w_z$$

By taking into consideration that the vehicle is symmetrical in (x,z) plane, the mathematical model of the bicycle (Fig.4) can be used, in which δ_f is the front wheel steering angle, and δ_r is the rear wheel steering angle; $2F_{xf}$ - is lumped longitudinal force at front wheel; $2F_{xr}$ - is lumped longitudinal force at rear wheel; $2F_{yf}$ - is lumped lateral force at front wheel; $2F_{yr}$ - is lumped lateral force at rear wheel; ψ -heading angle.

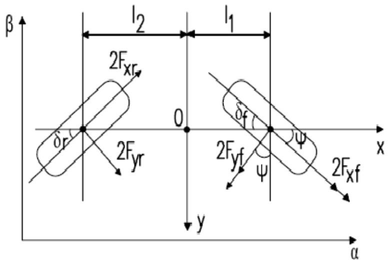

Fig.4. 2DOF Bicycle model of the electric vehicle and the velocity of the electric vehicle in (α, β) fixed coordinates system.

In case of small angular displacements, the forces added along the *y* axis, conduct to the following expression:

$$(2)\quad \begin{aligned} m a_y &= 2F_{xf}\sin\delta_f + 2F_{yf}\cos\delta_f + 2F_{yr}\cos\delta_r \\ &- 2F_{xr}\sin\delta_r \cong 2F_{yf} + 2F_{yr} \end{aligned}$$

where the lateral acceleration component is calculated as:

$$(3)\quad a_y = \dot{v}_y + v_x \dot{\theta} = \ddot{y} + \dot{x} w_z$$

$w_z = \dot{\theta}$, the yaw velocity around z-axis

In case of small angular variations, by adding the torques toward z axis (Fig.4), the yaw equation of motion is obtained.

$$(4)\quad \begin{aligned} J_z \frac{dw_z}{dt} &= 2F_{yf} l_1 \cos\delta_f + 2F_{xf} l_1 \sin\delta_f - 2F_{yr} l_2 \cos\delta_r \\ &+ 2F_{xr} l_2 \sin\delta_r \cong 2l_1 F_{yf} - 2l_2 F_{yr} \end{aligned}$$

where J_z is moment of inertia of vehicle on z-axis.

2.1. Determinations of the tires slip angles, α_f and α_r (Fig.5)

Taking into consideration the following notations: the tire slip angles (α_f , front; α_r -rear); the cornering stiffness coefficients: of the front tires - $C_{\alpha f} = \frac{dF_{yf}}{d\alpha_f}$ and of the rear tires $C_{\alpha r} = \frac{dF_{yr}}{d\alpha_r}$.

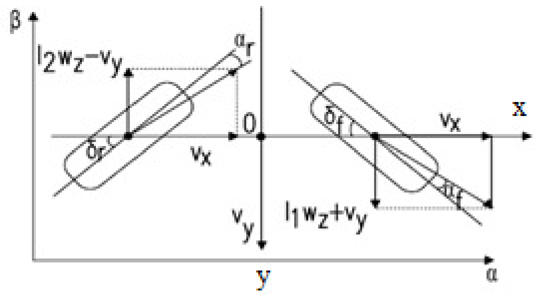

Fig.5. Tires slip angles representation

The following assumptions are made:

$$(5)\quad C_{\alpha_f} = C_{\alpha_r} = ct,$$

and are determined experimentally.

From Fig. 5, supposing the small angle variations, the tire slip angles (α_f , front; α_r -rear) are determined and taking into consideration the above mentioned equation, the equations of lateral forces becomes:

$$(6)\quad F_{yf} = C_{\alpha_f}\left(\delta_f - \frac{l_1 w_z + \dot{y}}{\dot{x}}\right),$$

$$F_{yr} = C_{\alpha_r}\alpha_r == \delta_r + \frac{l_2 w_z - \dot{y}}{\dot{x}}$$

where $v_y = \dot{y}$ and $v_x = \dot{x}$

By writing the y-axis force equilibrium equation, the lateral motion equation is obtained:

$$(7)\quad \begin{aligned} \ddot{y} &= -\left(\frac{2C_{\alpha f} + 2C_{\alpha r}}{m v_x}\right)\dot{y} + \left(-v_x - \frac{2C_{\alpha f} l_1 - 2C_{\alpha r} l_2}{m v_x}\right) w_z \\ &+ \frac{2C_{\alpha f}}{m}\delta_f + \frac{2C_{\alpha r}}{m}\delta_r. \end{aligned}$$

The yaw motion equation is obtained by using the F_{yf} and F_{yr} expressions:

$$\overset{\circ}{w}_z = -\left(\frac{2l_1C_{\alpha f} - 2l_2C_{\alpha r}}{y_z v_x}\right)\overset{\circ}{y} - \left(\frac{2l_1^2C_{\alpha r} + 2l_2^2C_{\alpha r}}{J_z v_x}\right)w_z + \frac{2l_1C_{\alpha f}}{J_z}\delta_f - \frac{2l_2C_{\alpha r}}{J_z}\delta_r. \quad (8)$$

The vehicle system response must be obtained in fixed coordinate system. Therefore, from Fig.4, by taking into consideration the small angular variations assumption, the velocity of the electric vehicle in (α,β) fixed coordinates system could be obtained:

$$\overset{\circ}{\beta} = -\overset{\circ}{x}\sin\psi - \overset{\circ}{y}\cos\psi \cong -v_x\psi - \overset{\circ}{y} \quad (9)$$

$$\overset{\circ}{\alpha} = \overset{\circ}{x}\cos\psi - \overset{\circ}{y}\sin\psi \quad (10)$$

For a sudden step lane change, the vehicle yaw rate is equal to the yaw rate error: $w_z = \dot{\psi}$ and it is assumed that: $\psi = \theta$.

By adequately choosing the state variables, the standard form of the state space mathematical model can be determined:

$$x_1 = \overset{\circ}{y},\ x_2 = \theta,\ x_3 = w_z,\ x_4 = \beta, \quad (11)$$

respectively:

$$\begin{bmatrix}\dot{x}_1\\ \overset{\circ}{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}a_{11} & 0 & a_{13} & 0\\ 0 & 0 & 1 & 0\\ a_{31} & 0 & a_{33} & 0\\ -1 & -v_x & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}b_{11} & b_{12}\\ 0 & 0\\ b_{31} & b_{32}\\ 0 & 0\end{bmatrix}\begin{bmatrix}\delta_f\\ \delta_r\end{bmatrix} \quad (12)$$

where,

$$a_{11} = \frac{-2C_{\alpha_f} - 2C_{\alpha_r}}{mv_x},\quad a_{13} = v_x - \frac{2C_{\alpha_f}l_1 - 2C_{\alpha_r}l_2}{mv_x},$$

$$a_{31} = \frac{-2l_1C_{\alpha_f} + 2l_2C_{\alpha_r}}{J_z v_x}\quad a_{33} = \frac{-2l_1C_{\alpha_f} + 2l_2C_{\alpha_r}}{J_z v_x},$$

$$b_{11} = \frac{2C_{\alpha_f}}{m},\quad b_{12} = \frac{2C_{\alpha_r}}{m},\quad b_{31} = \frac{2l_1C_{\alpha_f}}{J_z} \text{ and}$$

$$b_{32} = \frac{2l_2C_{\alpha_r}}{J_z}$$

3. OPTIMAL CONTROLLER DESIGN

The control input of lane-keeping controller is determined by using linear quadratic control theory. All state variables of vehicle are fed back to determine the current steering wheel angles.

The objective for the electric vehicle control is to achieve an asymptotically stable system response for a step lane change maneuver.

A linear quadratic regulator based controller is employed in order to obtain an asymptotic stable response for a controllable state space model.

3.1. State feedback optimization problem. The problem with free final time, and free final state

The discussed dynamic system () is linear, time invariant:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\cdot\mathbf{x}(t) + \mathbf{B}\cdot\mathbf{u}(t) \text{ cu } \mathbf{x}(t_0) = \mathbf{x}_0 \quad (13)$$

In order to optimize the system, the quadratic cost function or performance index is introduced:

$$J = \frac{1}{2}\int_{t_0}^{\infty}\left[\langle\mathbf{x}(t),\mathbf{Qx}(t)\rangle + \langle\mathbf{u}(t),\mathbf{Ru}(t)\rangle\right]dt \quad (14)$$

where **Q**, **R** are weighting matrices, the final state **x**(T) is unspecified, and the free final time, $T\to\infty$.

The optimal control problem consists in determining optimal control so that performance index (2) has to be minimized.

The initial and final conditions specifications: it is assumed that the system starts from zero initial state at time $t = 0$, without specifying the final state.

In order to solve the optimal control problem, the Hamiltonian is introduced as followed:

$$H[\mathbf{x}(t),\mathbf{p}(t),\mathbf{u}(t),t] = \frac{1}{2}[<\mathbf{x}(t),\mathbf{Qx}(t)> + <\mathbf{u}(t),\mathbf{Ru}(t)>] + <\mathbf{p}(t),[\mathbf{Ax}(t)+\mathbf{Bu}(t)]> \quad (15)$$

Candidates to optimal solutions are obtained by canceling the first order derivative of the Hamiltonian (15). Therefore, the optimal control or minimum point could be obtained from the following equation:

$$(16)\ \frac{\partial H}{\partial \mathbf{u}}\left[\mathbf{x}^*(t),\mathbf{p}^*(t),\mathbf{u}^*(t),t\right]=0$$

Thereby, optimal control, $\mathbf{u}^*$, is determined from:

$$(17)\ \mathbf{u}(t)=-\mathbf{R}^{-1}\mathbf{B}'\mathbf{p}(t)$$

In order to determine the costate vector p(t), it is obtained differential equation of costate vector:

$$(18)\ \dot{\mathbf{p}}(t)=-\frac{\partial H}{\partial \mathbf{x}(t)}$$

or

$$(19)\ \dot{\mathbf{p}}(t)=-\mathbf{Q}\mathbf{x}(t)-\mathbf{A}'\mathbf{p}(t)$$

Replacing the optimal control (17) in the dynamical system (13), the following equation is obtained:

$$(20)\ \dot{\mathbf{x}}(t)=\mathbf{A}\cdot\mathbf{x}(t)-\mathbf{B}\cdot\mathbf{R}^{-1}\mathbf{B}'\mathbf{p}(t)$$

or as:

$$(21)\ \dot{\mathbf{x}}=\mathbf{A}\cdot\mathbf{x}(t)-\mathbf{S}\mathbf{p}(t),$$

in which the **S** weight matrix is denoted as:

$$(22)\ \mathbf{S}=\mathbf{B}\cdot\mathbf{R}^{-1}\mathbf{B}'.$$

Thus, it is obtained reduced canonical system (23), linear, invariant, 2n dimension.

$$(23)\ \begin{bmatrix}\dot{\mathbf{x}}(t)\\ \dot{\mathbf{p}}(t)\end{bmatrix}=\begin{bmatrix}\mathbf{A} & -\mathbf{S}\\ -\mathbf{Q} & -\mathbf{A}'\end{bmatrix}\begin{bmatrix}\mathbf{x}(t)\\ \mathbf{p}(t)\end{bmatrix}$$

To solve this system it is necessary a number of 2n frontier conditions. A number of n conditions are obtain from initial condition $\mathbf{x}(t_0)$. The other n conditions are obtained from transversality condition of cost vector:

$$(24)\ \lim_{t\to\infty}\mathbf{p}(t)=0$$

System controllability is checked through the controllability matrix, the rank of controllability matrix is equal to the order of the dynamic system.

The dynamic system (13) is controllable and by a proper choice of weighting matrix **R**>0 and **Q**≥0, can be assert that optimal solution exists and is unique (Athans, 1966):

$$(25)\ \mathbf{u}(t)=-\mathbf{R}^{-1}\mathbf{B}'\hat{\mathbf{K}}\mathbf{x}(t)$$

where $\hat{\mathbf{K}}$ is a constant matrix, positively defined, solution of a nonlinear matrix algebraic Riccati equation (MARE):

$$(26)\ -\hat{\mathbf{K}}\mathbf{A}-\mathbf{A}'\hat{\mathbf{K}}+\hat{\mathbf{K}}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}'\hat{\mathbf{K}}-\mathbf{Q}=0$$

Optimal control law (25) assures an asympltotically stable system and generates the optimal trajectory:

$$(27)\ \dot{\mathbf{x}}(t)=\left(\mathbf{A}-\mathbf{B}\mathbf{R}^{-1}\mathbf{B}'\hat{\mathbf{K}}\right)\mathbf{x}(t),\quad \mathbf{x}(0)=0,$$

minimizing the functional cost, the minimum is given by:

$$(28)\ J^*[\mathbf{x}(t)]=\frac{1}{2}\left\langle \mathbf{x}(t),\hat{\mathbf{K}}\mathbf{x}(t)\right\rangle$$

for every $\mathbf{x}(t)$.

The control regulates the state trajectories close to the origin without excessive control demand.
The state feedback gains are determined such as the functional cost (14) is minimized.

4. IMPLEMENTATION OF THE OPTIMAL CONTROL

In order to show the effectiveness of the proposed solution, the following electric vehicle data is considerate:

4.1. Vehicle data

$$(29)\ C_{\alpha_f}=C_{\alpha_r}=30000\frac{N}{rad},\ l_1=1.25m,\ l_2=1.27m$$

$$m=1280kg,\ J_z=2200kg\cdot m^2,\ v_x=21.3\frac{m}{s}$$

4.2. Dynamic system

Dynamic system under control is described by equation (13). The output vector consists of the individual lateral motion of the vehicle, $\dot{y}(t)$, and the yaw angle, $\theta(t)$.

4.3. Choosing of the weighting matrices

Q matrix: $Q=\operatorname{diag}\{1\ 1\ 1\ 1\}$

is chosen such that the transients in electric drive to take place with minimum energy losses in the stator windings and with optimal kinetic energy distribution.

R matrix: $R=\operatorname{diag}\{1\ 1\}$

R matrix is intended to maintain the optimal control within acceptable limits.

4.4. Initial and final conditions

The initial conditions consist of initial time t_0 and the initial state x_0. For the particular case of the starting these conditions are:

$$(30)\ t_0 = 2[s];\ x(t_0) = \begin{bmatrix} \dot{y}(t_0) \\ \theta(t_0) \\ w_z(t_0) \\ \beta(t_0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Final conditions consist of the final time and free final state

$$(31)\ t_f = 6[s];\ x_f = \begin{bmatrix} \dot{y}(t_f) \\ \theta(t_f) \\ w_z(t_f) \\ \beta(t_f) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

It is noted that the actual state values are established naturally by using the adequate mathematical model of the process, therefore the optimal control problem being with free final state.

The state vector of the vehicle system is: $x(t) = \begin{bmatrix} \dot{y}(t) \\ \theta(t) \\ w_z(t) \\ \beta(t) \end{bmatrix}$, and the control vector is given by the front and rear wheel steering angles $u^*(t) = \begin{bmatrix} \delta_f^*(t) \\ \delta_r^*(t) \end{bmatrix}$, taking into consideration a unitary transmission rate, and neglecting the servomotor dynamics.

Optimal control solution is given by (25). Therefore, the steering angles are calculated as a state feedback control input.

The $\hat{\mathbf{K}}$ matrix is determined by replacing the corresponding matrices A, B, R, Q in (26) :

$$\hat{\mathbf{K}} = \begin{matrix} 0.1082 & 0.1279 & 1.1843 & 0.7593 \\ 0.1279 & 0.1082 & 0.7593 & 1.1843 \end{matrix}$$

In order to cancel the steady-state error an adequate reference filter has been, the voltage control has been obtained from:

$$(32)\ u_{sq}^* = -R^{-1}B'\hat{K}x + vr$$

The v constant must be inserted in order to obtain a zero steady state error. The considered steady state control error is as follow:

$$(33)\ e_s = \lim_{t\to\infty}\big(r(t) - y(t)\big) = r_s - \lim_{t\to\infty} y(t)$$
$$= r_s - c^T \underbrace{\lim_{t\to\infty} x(t)}_{=x_s}$$

The stationary solution x_s results from the following:

$$(34)\ 0 = (A - bk^T)x_s + bvr_s$$
$$(35)\ x_s = -(A - bk^T)^{-1}bvr_s$$

The steady state error:

$$(36)\ e_s = r_s + c^T(A - bk^T)^{-1}bvr_s$$
$$(37)\ \frac{e_s}{r_s} = 1 + c^T(A - bk^T)^{-1}bv$$

Therefore, the steady state error does not appear if the following equation is valid:

$$(38)\ v = -\frac{1}{c^T(A - bk^T)^{-1}b}$$

The structural block diagram of the entire drive control is shown in Fig. (6):

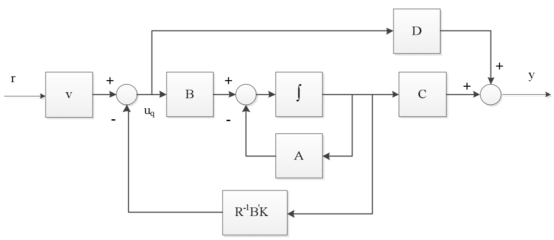

Fig.6. Structural block diagram of the optimal control of linearized system

4.5. Validation of the optimal solution by numerical simulation

calfaf = 30000
calfar=calfaf
l1 = 1.25
l2 = 1.27
m = 1380
Jz = 2200
vx=21.3

The numerical values of the specific matrices (A, B) of the dynamic system are as follows:

A =

-4.0825	0	-21.2592	0
0	0	1.0000	0
0.0256	0	-4.0658	0

```
  -1.0000  -21.3000         0         0

B =
  43.4783   43.4783
        0         0
  34.0909  -34.6364
        0         0
```

The controllability matrix: `Qr=ctrb(A,B)`

Qr = 1.0e+004 *

```
 Columns 1 through 6

  0.0043    0.0043   -0.0902    0.0559    0.6606   -0.5299
       0         0    0.0034   -0.0035   -0.0137    0.0142
  0.0034   -0.0035   -0.0137    0.0142    0.0536   -0.0563
       0         0   -0.0043   -0.0043    0.0176    0.0179

 Columns 7 through 8

 -3.8364    3.3597
  0.0536   -0.0563
 -0.2010    0.2152
 -0.3678    0.2276
```

The rank of the matrix Q_r can be find as:

`rank(Qr):` ans = 4

The weighting matrices are chosen as in Section (4.3).

The optimal state feedback gain, as the solution of the matrix Riccati algebraic equation:

```
K =
   0.5862    6.2017    0.6624   -0.9401
   0.7389   -3.3525   -0.8676    0.3409

S =
   0.2979   -0.9401
   0.9592    0.3409
```

Taking into consideration that the regulator-based closed loop system can be described by the following matrices:

```
Ac = A-B*K;
Bc=[0 1
    0 0
    1 0
    0 0]
```

The entire system is embedded in Matlab as:

sys = ss(Ac, Bc, Cc, Dc);

The solutions of the characteristic equation are as follows:

poles =eig(Ac)

```
 -55.9664 + 6.7568i
 -55.9664 - 6.7568i
  -3.2296 + 3.1087i
  -3.2296 - 3.1087i
```

For a stable system, the real part of poles must all be less than zero. It could be noted that an asymptotically stable system responses can be obtained, taking into consideration that the real part of the system’s closed loop poles are situated in the left side of the s-plane.

5. SIMULATION RESULTS

To shows the efficacity of the proposed control the step-lane maneuveur test has been used and the system’s responses have been obtained (Fig. 7 to Fig.9).

The lane change reference is similar with a step signal. It could be noted that the stable behaviour of the vehicle output response (Fig.7) was obtained by applying the optimal control (25).

In order to obtain a zero steady-state error, an adequate input filter *v* has been applied (38). Also, by introducing the optimal controller, the state variables are limited due to the proper choice of weighting matrices, and by minimizing the quadratic performance index it ensures energy savings.

The performances of the proposed control are underlined in Fig. 7 to Fig. 9, considering a step lane reference for the vehicle system.

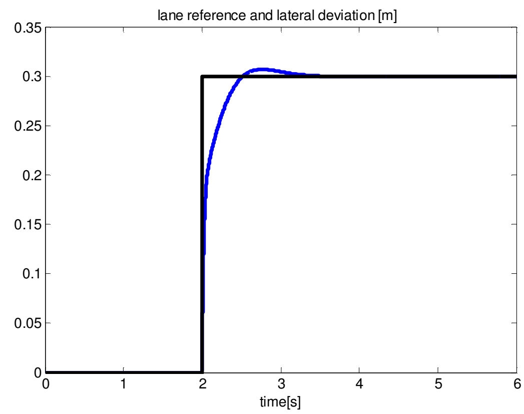

Fig.7 The vehicle lateral desired lane and the actual vehicle response to the applied step reference

The vehicle system behavior to the applied step reference (0.3m applied at 2s after simulation running) is shown in Fig.7. It could be underlined, that by applying optimal control (25) the asymptotically stable system is obtained.

The optimal control vector is given by the front (Fig.8) and rear (Fig.9) wheel steering angles

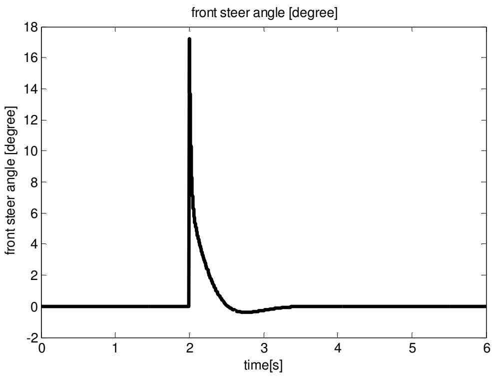

Fig.8 The front steer angle response of the vehicle

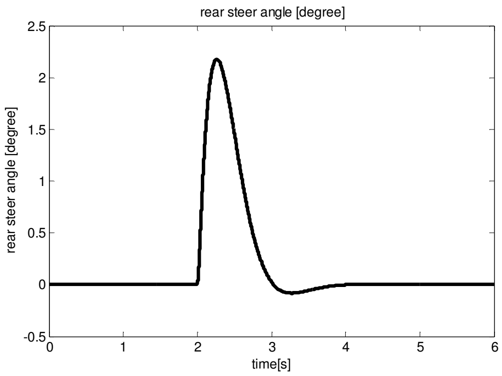

Fig.9 The rear steer angle vehicle response

6. CONCLUSIONS

In this paper, a suitable optimal control for electric vehicle system stabilization purposes has been proposed.

Considering a direct mechanical contact of the steering wheel, for improving handling and stability of vehicle the performance of the proposed control is evaluated under a step lane maneuver by numerical simulation. The optimal control regulates the system responses (state trajectories of the state vector close to origin without excessive control demand) and assure an asymptotically vehicle system such that the performance index (14) is minimized. The obtained results show that the feasibility of the proposed control solution.

In order to implement the multivariable optimal controller the lateral dynamics of the vehicle were considered. The entire system was simulated in Matlab/Simulink environment. The closed loop optimal vehicle system is a stable one.

Fast perturbation influence rejection can be observed at t = 2s, in a relatively short time. The filter on the reference vector leads to a zero steady state error. Therefore, the imposed objectives were successfully achieved.

7. ACKNOWLEDGEMENTS

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